nyt

[entry]nyt/global/
3.36pt

The Complexity of Non-Gaussian Quantum Circuits

Robert Booth

June 11th, 2018

What is a quantum computer?

classical bit : 0 or 1

What is a quantum computer?

classical bit : 0 or 1

"quantisation"

quantum bit : $\vec{\psi} = \alpha\vec{\mathbf{0}} + \beta\vec{\mathbf{1}}$

$$= \alpha\,|0\rangle + \beta\,|1\rangle$$

What is a quantum computer?

classical bit : 0 or 1

“quantisation”

quantum bit : $\vec{\psi} = \alpha\vec{\mathbf{0}} + \beta\vec{\mathbf{1}}$

$$= \alpha\,|0\rangle + \beta\,|1\rangle$$

where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$

What is a quantum computer?

classical bit : 0 or 1

"quantisation"

reading the state of a qubit:

$$\mathrm{Prob}(0|\vec{\psi}) = |\alpha|^2$$
$$\mathrm{Prob}(1|\vec{\psi}) = |\beta|^2$$

quantum bit : $\vec{\psi} = \alpha\vec{\mathbf{0}} + \beta\vec{\mathbf{1}}$

$$= \alpha|0\rangle + \beta|1\rangle$$

where $\alpha, \beta \in \mathbb{C}$ and $|\alpha|^2 + |\beta|^2 = 1$

What is a quantum computer?

classical string : 001100100

classical string : 001100100

quantum state : $|001100100\rangle$ encoded by 8 qubits

What is a quantum computer?

classical string : 001100100

Linear combinations of strings:

$|\psi\rangle = \alpha\,|00\rangle + \beta\,|11\rangle$

quantum state : $|001100100\rangle$ encoded by 8 qubits

Quantum logic gates

- gates are *linear* (even unitary):

$$|0\rangle \longmapsto \alpha\,|0\rangle + \beta\,|1\rangle$$

Quantum logic gates

- gates are *linear* (even unitary):

$$|0\rangle \longmapsto \alpha |0\rangle + \beta |1\rangle$$

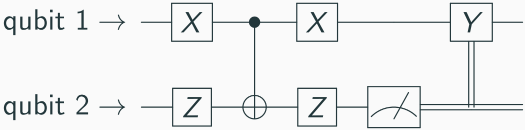
- universal set : $\{H, Z, C_Z, T\}$

Quantum logic gates

- gates are *linear* (even unitary):

$$|0\rangle \longmapsto \alpha |0\rangle + \beta |1\rangle$$

- universal set : $\{H, Z, C_Z, T\}$

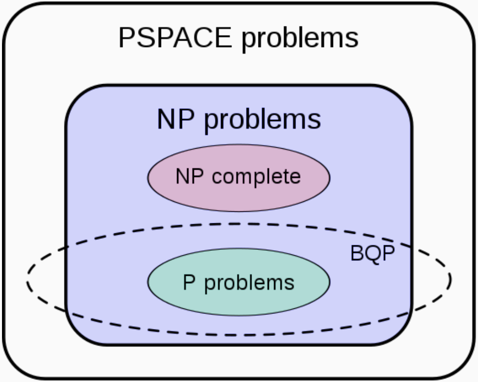


Why quantum computation?

	Classical	Quantum
Prime Factoring	$2^{\tilde{O}(n^{1/3})}$	$\tilde{O}(n^3)$
Search	$\Theta(n)$	$\Theta(\sqrt{n})$
Discrete Logarithm	$O(n2^n)$	$O(n\log(n))$
Sparse Linear Systems	$O(kn)$	$O(k^2 \log n)$

See `https://math.nist.gov/quantum/zoo/` for more examples

Complexity classes



Why are quantum computers faster?

A number of suggestions:

Why are quantum computers faster?

A number of suggestions:

- parallelism

Why are quantum computers faster?

A number of suggestions:

- parallelism
- interference

Why are quantum computers faster?

A number of suggestions:

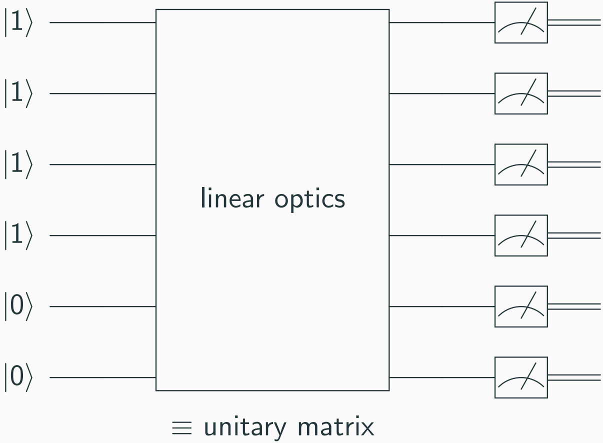
- parallelism
- interference
- entanglement

Why are quantum computers faster?

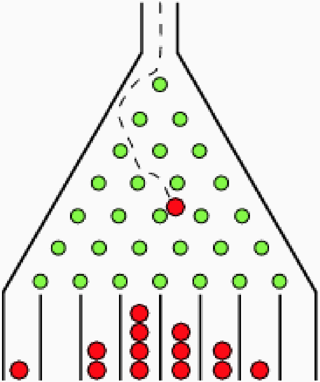
A number of suggestions:

- parallelism
- interference
- entanglement

Nobody knows...


$|1\rangle$
$|1\rangle$
$|1\rangle$
$|1\rangle$
$|0\rangle$
$|0\rangle$
linear optics
$\equiv$ unitary matrix

Sampling problems (cont.)



Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)

Simulation algorithms:

Some previous results

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)

Simulation algorithms:

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)

Simulation algorithms:

- Gottesman-Knill theorem (**gottesman˙heisenberg˙1998**)

Some previous results

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)

Simulation algorithms:

- Gottesman-Knill theorem (**gottesman˙heisenberg˙1998**)

$$\text{gate set: } \{H, Z, C_{NOT}\}$$

Some previous results

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)

Simulation algorithms:

- Gottesman-Knill theorem (**gottesman˙heisenberg˙1998**)
- Efficient simulation of $\log(T)$ circuits (**bravyi˙improved˙2016**)

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)

Simulation algorithms:

- Gottesman-Knill theorem (**gottesman˙heisenberg˙1998**)
- Efficient simulation of $\log(T)$ circuits (**bravyi˙improved˙2016**)

gate set: $\{H, Z, C_{NOT}\} + T$

Continuous Variables

- **qubit :** $\langle\, |0\rangle\,, |1\rangle\,\rangle$

- **qubit :** $\langle\, |0\rangle\,, |1\rangle\,\rangle$
- **qumode :** $\langle\, |q\rangle \mid q \in \mathbb{R} \rangle$

- **qubit :** $\langle\, |0\rangle\,, |1\rangle\,\rangle \rightsquigarrow |\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- **qumode :** $\langle\, |q\rangle \mid q \in \mathbb{R} \rangle$

Continuous Variables

- **qubit :** $\langle\, |0\rangle, |1\rangle \,\rangle \rightsquigarrow |\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- **qumode :** $\langle\, |q\rangle \mid q \in \mathbb{R} \rangle \rightsquigarrow |\psi\rangle = \int_{\mathbb{R}} f(q)\,|q\rangle\,\mathrm{d}q$

- **qubit :** $\langle\, |0\rangle\,, |1\rangle\,\rangle \rightsquigarrow |\psi\rangle = \alpha\,|0\rangle + \beta\,|1\rangle$

- **qumode :** $\langle\, |q\rangle \mid q \in \mathbb{R} \rangle \rightsquigarrow |\psi\rangle = \int_{\mathbb{R}} f(q)\,|q\rangle\,\mathrm{d}q$

$$\left(\quad \text{s.t.} \quad \int_{\mathbb{R}} |f(q)|^2\,\mathrm{d}q = 1 \right)$$

- **DV :** $\{H, Z, C_{NOT}\} + T$

- **CV :** $\{F, C_Z, X(s), P(\eta) \mid s, \eta \in \mathbb{R}\}$
  $+$ *any* non-Gaussian gate

Simulation algorithms:

- Gottesman-Knill theorem (**gottesman˙heisenberg˙1998**)
- Efficient simulation of $\log(T)$ circuits (**bravyi˙improved˙2016**)

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)

Simulation algorithms:

- Gottesman-Knill theorem (**gottesman˙heisenberg˙1998**)
- Efficient simulation of $\log(T)$ circuits (**bravyi˙improved˙2016**)
- Simulation of Gaussian CV circuits (**bartlett˙efficient˙2002**)

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)

Some previous results

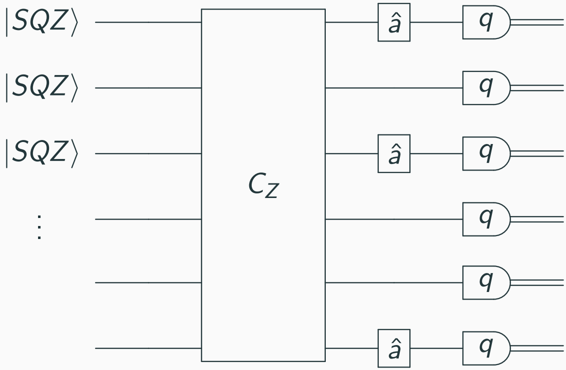
Simulation algorithms:

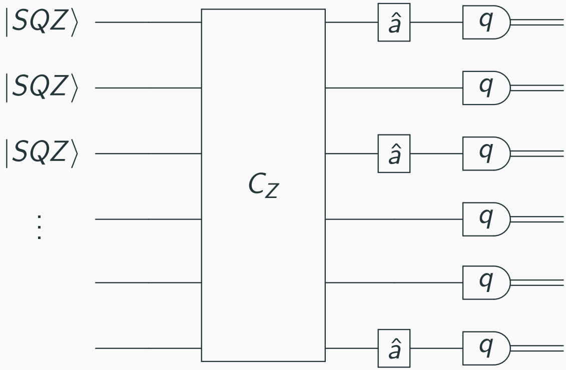
- Gottesman-Knill theorem (**gottesman˙heisenberg˙1998**)
- Efficient simulation of $\log(T)$ circuits (**bravyi˙improved˙2016**)
- Simulation of Gaussian CV circuits (**bartlett˙efficient˙2002**)

Hardness:

- BosonSampling and permanents of matrices (**aaronson˙computational˙2010**)
- IQP (**bremner˙classical˙2011**)
- Continuous variable sampling problems (**chabaud˙continuous-variable˙2017**)

Photon substractions





$\rightsquigarrow$ permanent of an $n_{\hat{a}} \times n_{\hat{a}}$ matrix

Ryser's algorithm : $O(2^n n^2)$

Further work

- Can this be extended to universal CV computation?
- How to quantify the "non-Cliffordness" of a given circuit?

Thanks for listening...