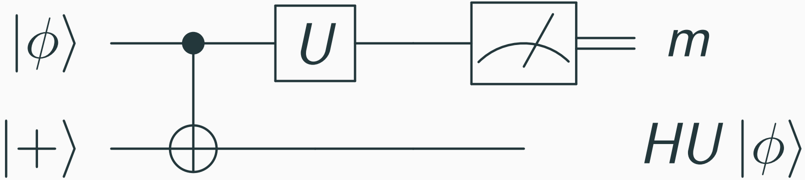
Flow Conditions for Continuous Variables Measurement-based Quantum Computation

Robert Booth

June 28th, 2019

MBQC

- Gate teleportation



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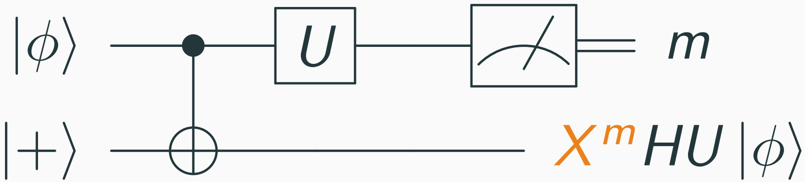

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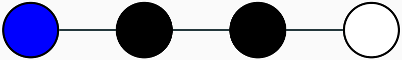

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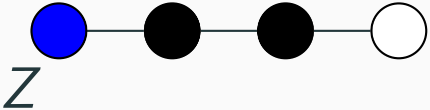

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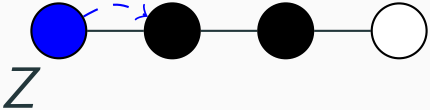

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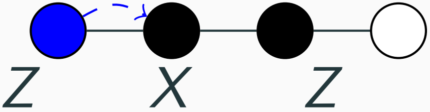

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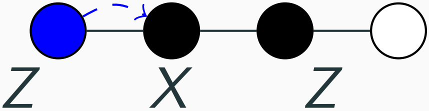

$$K_i = X_i \prod_{j \in G(i)} Z_j$$

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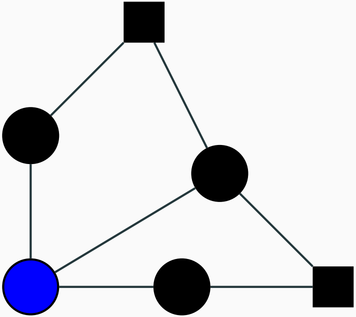

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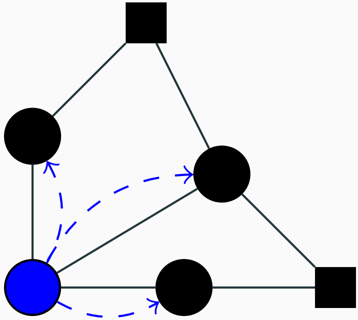

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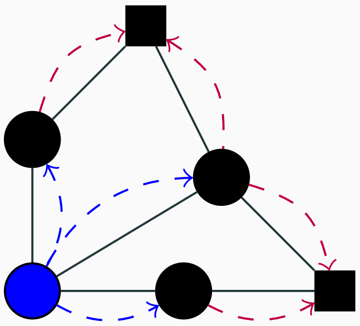
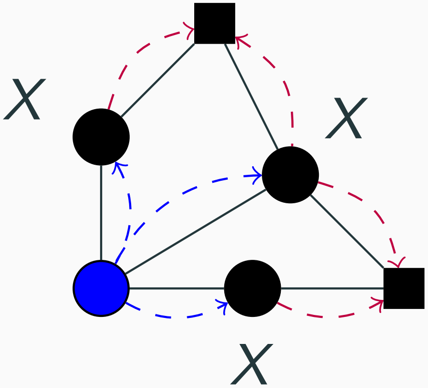
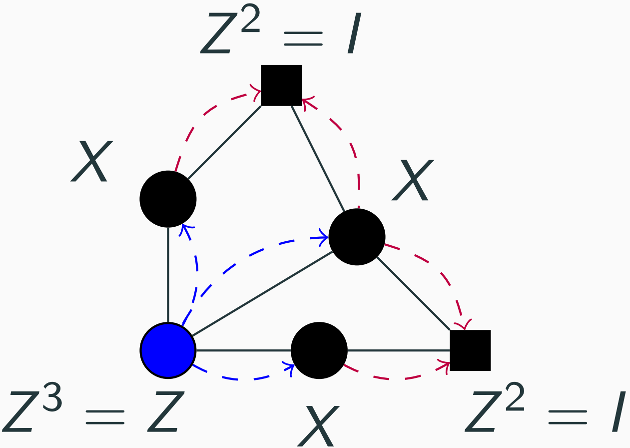
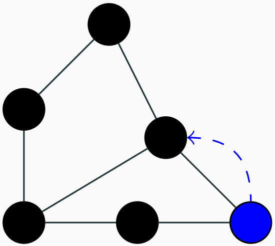
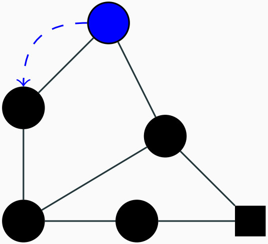
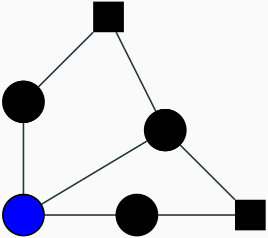


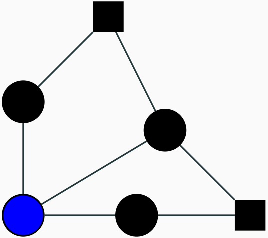

X
X
X


$Z^2 = I$
X
X
$Z^3 = Z$
X
$Z^2 = I$









- Linear equations:

$$A_i \vec{c} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{with} \quad \vec{c} \in \mathbb{Z}_2^n$$

- state space: qumodes $\rightsquigarrow \mathrm{L}^2(\mathbb{R})$, i.e.

$$\int_{\mathbb{R}} |f|^2 < \infty$$

A primer on CV

- state space: qumodes $\rightsquigarrow \mathrm{L}^2(\mathbb{R})$, i.e.

$$\int_{\mathbb{R}} |f|^2 < \infty$$

- key example: squeezed state $|\eta\rangle \rightsquigarrow$ a Gaussian of width $\frac{1}{2\eta^2}$ centered at $x = 0$

- gates are unitaries $\mathrm{L}^2(\mathbb{R})^{\otimes n} \to \mathrm{L}^2(\mathbb{R})^{\otimes n}$
- "standard" set:

$$\{\mathsf{F}, \exp(is\mathsf{Q}_j), \exp\left(is\mathsf{Q}_j^2\right), \exp\left(is\mathsf{Q}_j^3\right), \exp(i\mathsf{Q}_j\mathsf{Q}_k)\}$$

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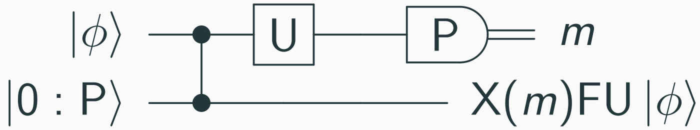
- Errors: $Z(s)$ corrected by $Z(-s)$
- Stabilisers: $K_i(s) = X_i(s) \prod_{j \in G(i)} Z_j(s)$
- Commutation rules are essentially the same!



$|\phi\rangle$
$|0:P\rangle$
U
P
m
$X(m)FU|\phi\rangle$

CV Flow

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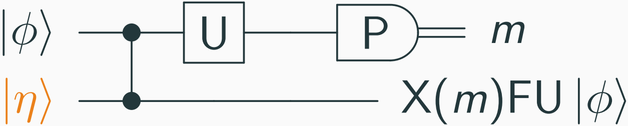
CV-MBQC



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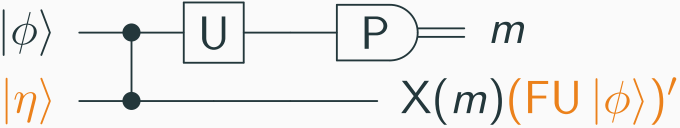

Convergence

- If G has CV-flow:

Convergence

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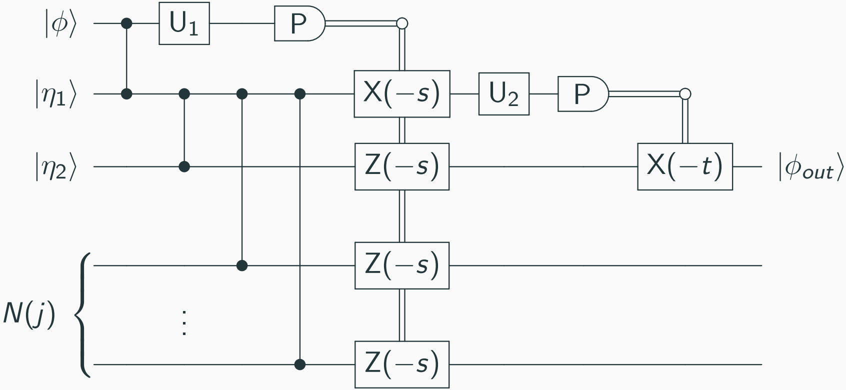
$$\lim_{\eta\to\infty} \left\| (\mathsf{O}_\eta - \mathsf{E}_\eta) \left|\phi\right\rangle \right\| = 0.$$

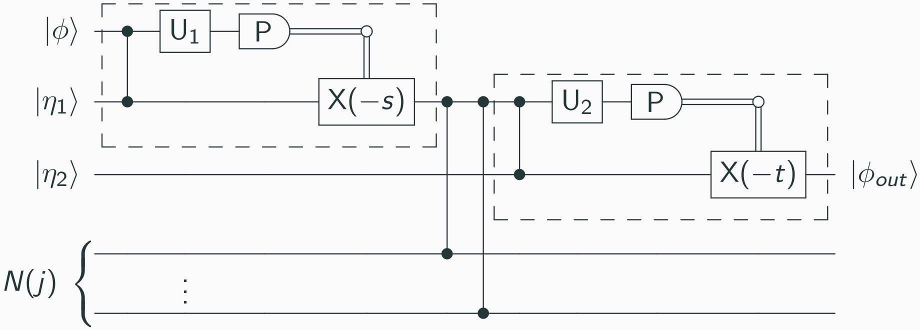
Circuit extraction

- Star Pattern Transformation



|ϕ⟩
|η1⟩
|η2⟩
N(j)
U1
P
X(−s)
U2
P
Z(−s)
Z(−s)
Z(−s)
X(−t)
|ϕout⟩

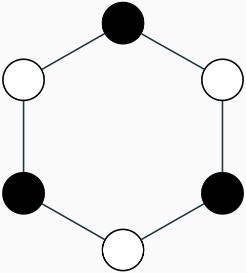
Circuit extraction

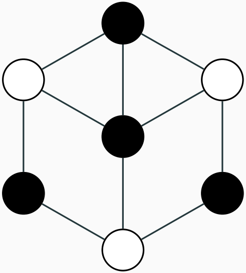


Circuit extraction

- Conclusion: For a graph with *flow*, the MBQC converges for "all" measurement choices if and only if $|I| = |O|$.

Are they equivalent?





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- Work ongoing to prove convergence with CV-flow.
- There is a poly-time algorithm for finding CV-flows if a graph has one.
- No attempt to quantify the scaling of convergence.
- It is possible to have a notion of flow for qudit MBQC using the same kind of construction.
- For qudits and CV, it is possible to “weight” the entangling operations.