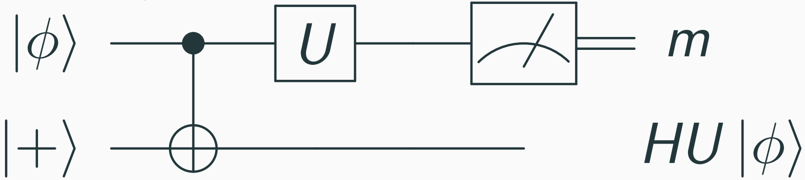
Flow Conditions for Continuous Variables Measurement-based Quantum Computation

Robert Booth, Damian Markham

July 17th, 2019

- Gate teleportation



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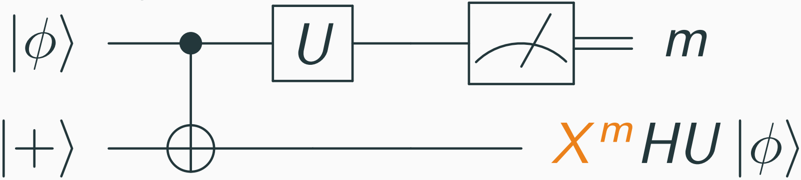
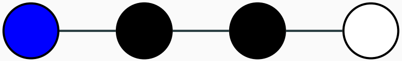


What is flow?



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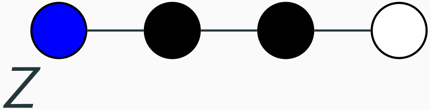

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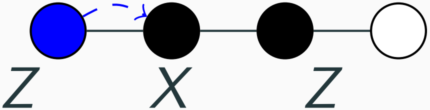

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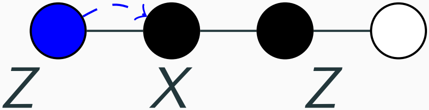

$$K_i = X_i \prod_{j \in G(i)} Z_j$$

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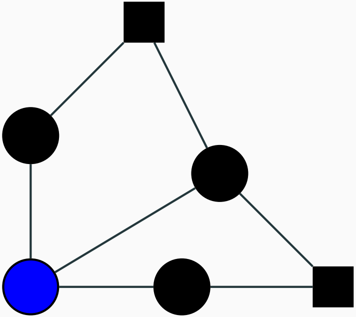

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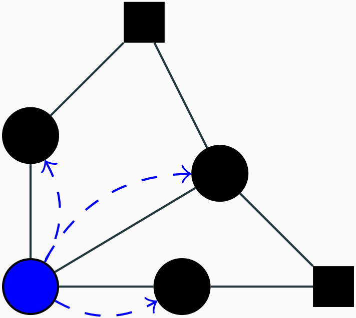

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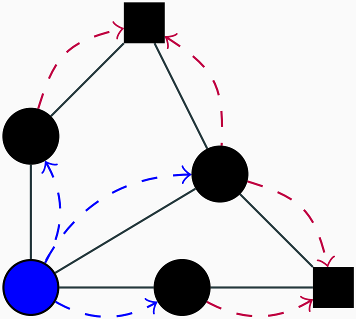

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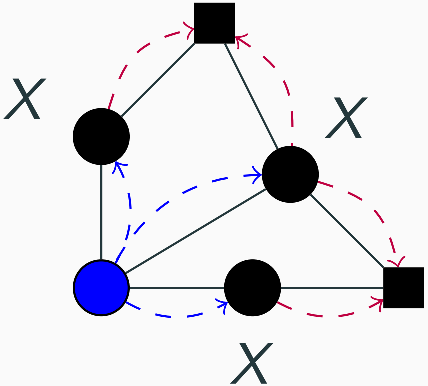

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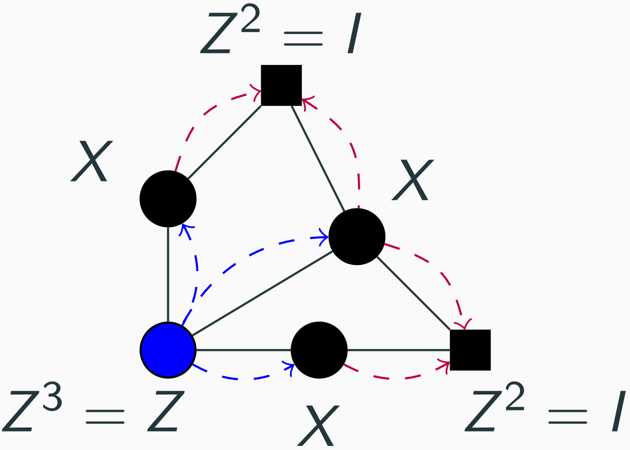

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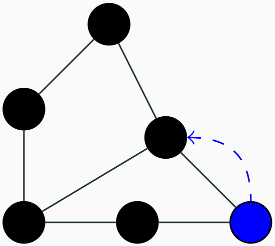

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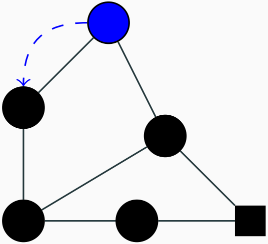

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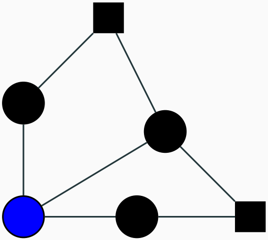

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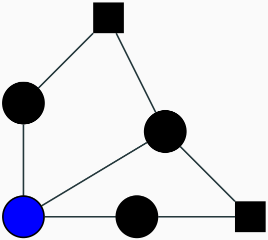

- Linear equations:

$$A_i \vec{c} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{with} \quad \vec{c} \in \mathbb{Z}_2^n$$

A primer on CV

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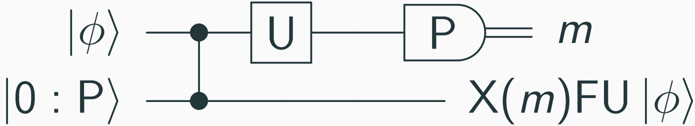
- key example: squeezed state $|\eta\rangle \rightsquigarrow$ a Gaussian of width $\frac{1}{2\eta^2}$ centered at $x = 0$


$|\phi\rangle$
$|0:\mathsf{P}\rangle$
U
P
m
$\mathsf{X}(m)\mathsf{F}\mathsf{U}\,|\phi\rangle$

CV Graph States

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- Errors: $Z(s)$ corrected by $Z(-s)$
- Stabilisers: $K_i(s) = X_i(s) \prod_{j \in G(i)} Z_j(s)$
- Commutation rules are essentially the same!

- DV:

$$A_i\vec{c} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{with} \quad \vec{c} \in \mathbb{Z}_2^n$$

CV Flow

- DV:

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- CV:

$$A_i\vec{c} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{with} \quad \vec{c} \in \mathbb{R}^n$$

CV-MBQC


$|\phi\rangle$
$|\eta\rangle$
U
P
m
$X(m)(FU|\phi\rangle)'$

Convergence

- If G has CV-flow:

$$\lim_{\eta\to\infty} \left\|(\mathsf{E}_\eta - \mathsf{O}_\eta)\,|\phi\rangle\right\| = 0.$$

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$$\lim_{\eta \to \infty} \| (\mathsf{E}_\eta - \mathsf{O}_\eta) |\phi\rangle \| = 0.$$

- If G has flow: $\mathsf{E}_\eta \to \mathsf{U}$ iff $|I| = |O|$.

Conclusion

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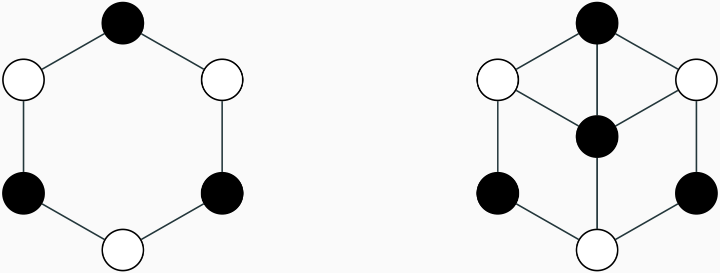
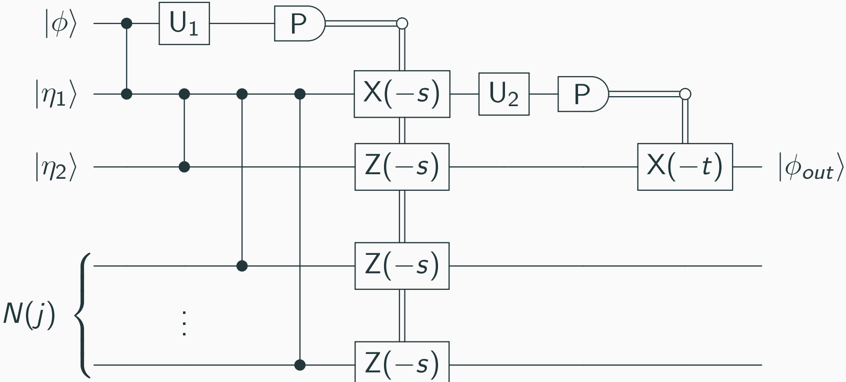
- There is a poly-time algorithm for finding CV-flows if a graph has one.
- No attempt to quantify the scaling of convergence.
- It is possible to have a notion of flow for qudit MBQC using the same kind of construction.
- For qudits and CV, it is possible to "weight" the entangling operations.

The End.

Are they equivalent?





$|\phi\rangle$
$|\eta_1\rangle$
$|\eta_2\rangle$
$N(j)$
U1
P
X(−s)
U2
P
Z(−s)
X(−t)
Z(−s)
Z(−s)
$|\phi_{out}\rangle$

Circuit extraction