Flow Conditions for Continuous Variable Measurement-based Quantum Computing

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Continuous variables: states

- "State space" $\to$ Hilbert space:

$$f \in \mathrm{L}^2(\mathbb{R}, \mathbb{C}) \iff f : \mathbb{R} \to \mathbb{C} \quad \text{s.t.} \quad \int_{x \in \mathbb{R}} |f(x)|^2 < \infty$$

$$\mathrm{Prob}(x \in E) = \int_{x \in E} |f(x)|^2$$

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- Countable orthonormal basis:

$$f = \sum_{n \in \mathbb{N}} c_n \xi_n \quad \text{where} \quad \xi_n \in \mathrm{L}^2(\mathbb{R}, \mathbb{C})$$

Continuous variables: transformations

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$$\left\{\mathsf{F}, \exp(i\alpha \mathsf{Q}), \exp\left(i\beta \mathsf{Q}^2\right), \exp\left(i\gamma \mathsf{Q}^3\right), \exp(iw\mathsf{Q} \otimes \mathsf{Q})\right\}$$

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$$\left\{ \mathsf{F}, \mathsf{U}, \mathsf{CZ}(w) \right\}$$

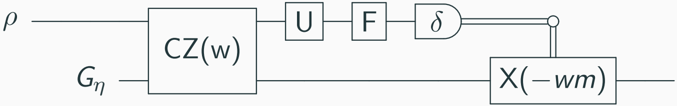
- Gates used in the MBQC procedure:

$$\mathsf{Z}(s) := \exp(-is\mathsf{P})$$
$$\mathsf{X}(s) := \exp(is\mathsf{Q})$$

Continuous variables: discussion

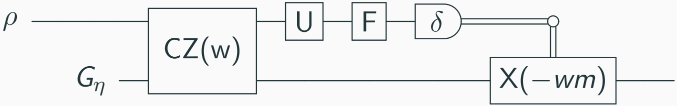
- CV embeds DV quantum computation
- Some CV systems: modes of the quantum electromagnetic field, supraconductors
- Experiments: high levels of entanglement

Measurement-based Quantum Computing



Measurement-based Quantum Computing

- Gate teleportation



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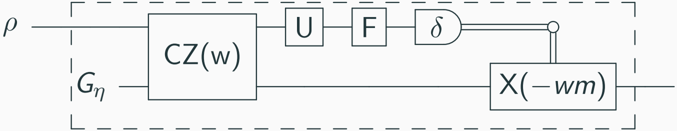

Measurement-based Quantum Computing

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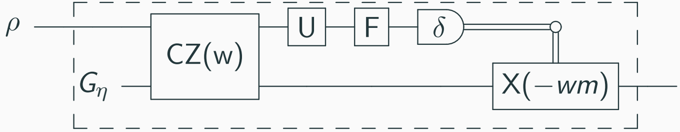


$$\updownarrow$$

$\sim \mathrm{FU}[\rho]$

- Main question: convergence?

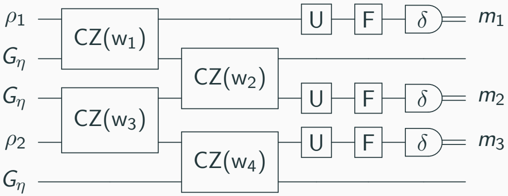
Measurement-based Quantum Computing (cont.)

Formally,

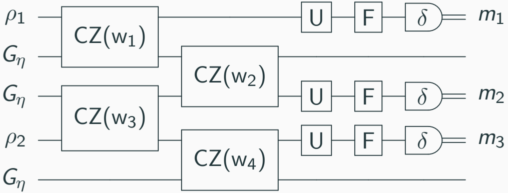
$$\lim_{\substack{\eta \to \infty \\ \delta \to 0}} \mathscr{C}_\eta^\delta[\rho] = \mathsf{S}(w)\mathsf{FU}[\rho]$$

MBQC protocols

MBQC protocols

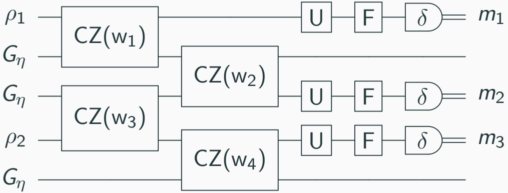


MBQC protocols



$$\lim_{\substack{\eta\to\infty\\ \delta\to 0}} \mathscr{M}_\eta^\delta[\rho_1 \otimes \rho_2]$$

MBQC protocols



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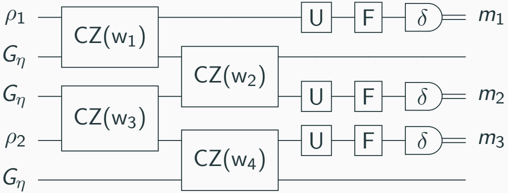
- Another question: runnability?

Graph states



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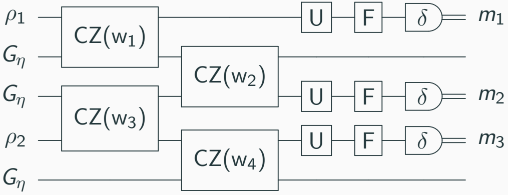

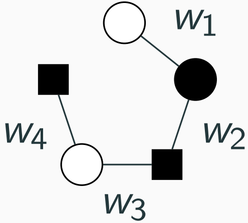

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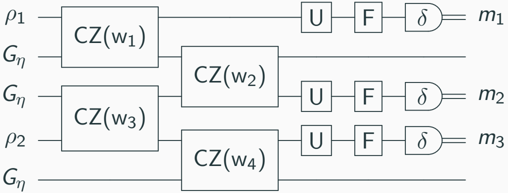

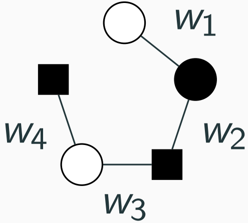

- Stabilisers: $K_i(s) = X_i(s) \prod_{j \in G(i)} Z_j(s)$

Graph states (cont.)

- Gate teleportation:

- Corrections?

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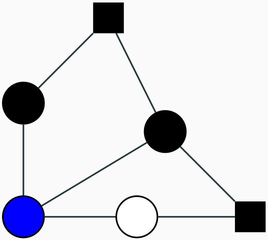
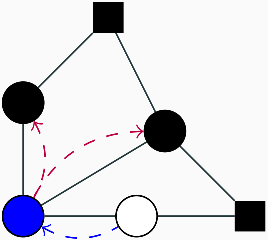


Graph states (cont.)

- Gate teleportation:

- Corrections?



Causal flow

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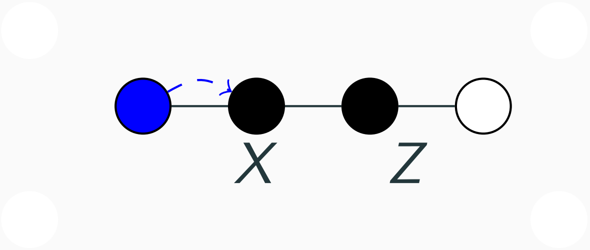
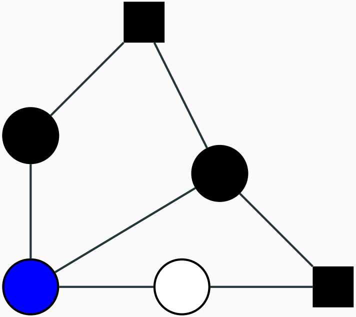


CV-flow



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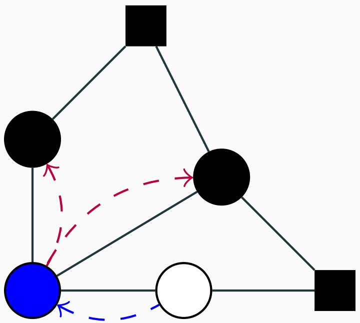

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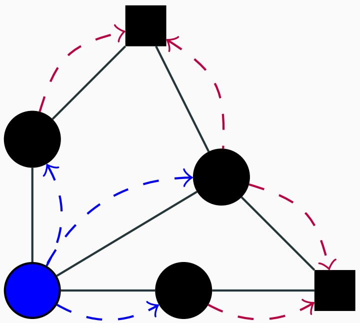

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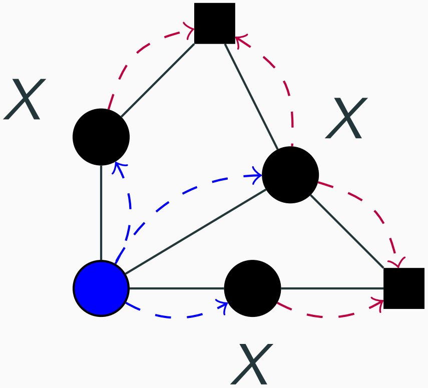

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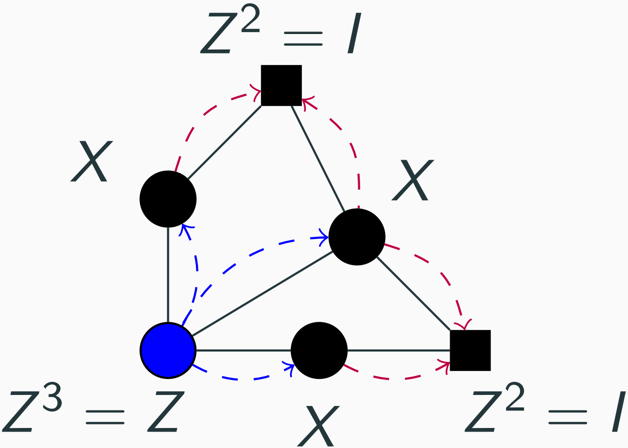

CV-flow (cont.)

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- Corrections sum:

$$\mathrm{Z}(a)\mathrm{Z}(b) = \mathrm{Z}(a+b)$$
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CV-flow (cont.)

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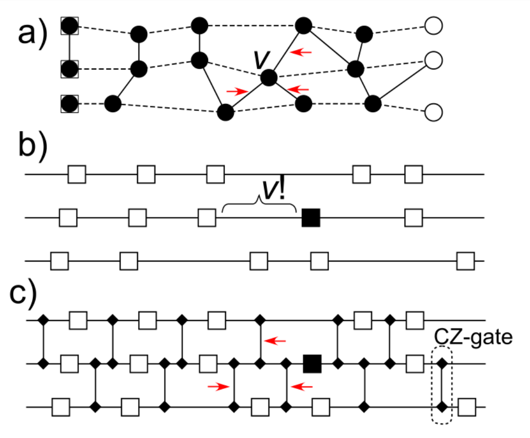
- Linear equations:

$$A_i\vec{c} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{with} \quad \vec{c} \in \mathbb{R}^n$$

Recap

Picture	Quantum Circuit	Graph State	Adjacency Matrices
Problem	Commute gates to make circuit runnable	Complete stabilisers	Solve linear equations

Convergence: causal flow



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[1] Jisho Miyazaki, Michal Hajdušek, and Mio Murao. "An Analysis of the Trade-off between Spatial and Temporal Resources for Measurement-Based Quantum Computation". In: *Physical Review A* 91.5 (May 2015), p. 052302.

Convergence: CV-flow

- Lemma: if a graph has CV-flow, then it can be split into layers, and each of these layers has a causal flow up to a *non-local* change of basis

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- Lemma: if a graph has CV-flow, then it can be split into layers, and each of these layers has a causal flow up to a *non-local* change of basis
- Lemma: this change of basis doesn't get in the way of the MBQC procedure
- Lemma: the endpoints of paths in subsequent layers match up
- Conclusion: the CV-flow procedure can be broken down into a sequence of SPT maps with intermediate changes of basis

Takeaway

- If a graph has a CV-flow, and the same number of input nodes as outputs, then there is an MBQC procedure and it converges.
- CV-flow is essentially causal flow in the "wrong" basis.
- We obtain an explicit circuit extraction scheme for the MBQC protocol when the graph state has CV-flow.
- (Poly-time algorithm for finding a CV-flow if one exists.)

Some open questions

1. A necessity theorem?
2. Cleaner proofs with distributions?
3. Generalisation to other groups?
4. Counting flows and Blind Quantum Computation?