Flow Conditions for Continuous Variable Measurement-based Quantum Computing

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November 19th, 2019

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reading the state of a qubit:

$$\mathrm{Prob}(0|\vec{\psi}) = |\alpha|^2$$
$$\mathrm{Prob}(1|\vec{\psi}) = |\beta|^2$$

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- Countable orthonormal basis:

$$f = \sum_{n\in\mathbb{N}} c_n \xi_n \quad \text{where} \quad \xi_n \in \mathrm{L}^2(\mathbb{R},\mathbb{C})$$

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$$\left\{\mathsf{F}, \exp(i\alpha \mathsf{Q}), \exp\left(i\beta \mathsf{Q}^2\right), \exp\left(i\gamma \mathsf{Q}^3\right), \exp(iw\mathsf{Q} \otimes \mathsf{Q})\right\}$$

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$$\left\{ \mathsf{F}, \mathsf{U}(\alpha, \beta, \gamma), \mathsf{CZ}(w) \right\}$$

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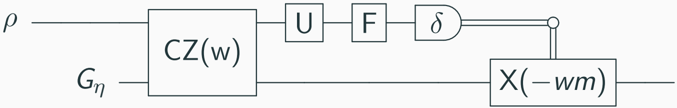
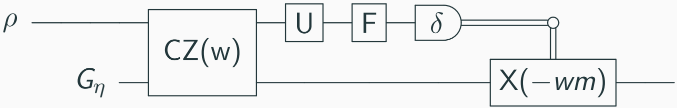
$$\Big(\text{Lie theory} \;\rightsquigarrow\; \exp : i\mathbb{R}[\mathsf{Q}, \mathsf{P}] \to \mathcal{U}(\mathcal{H})\Big)$$

Measurement-based Quantum Computing



Measurement-based Quantum Computing

- Gate teleportation



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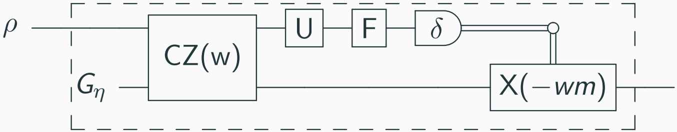

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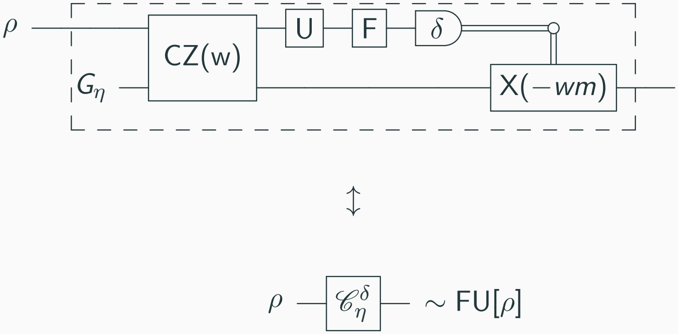

Measurement-based Quantum Computing (cont.)

Formally,

$$\lim_{\substack{\eta\to\infty \\ \delta\to 0}} \mathscr{C}_\eta^\delta[\rho] = \mathsf{S}(w)\mathsf{FU}[\rho]$$

Graph states

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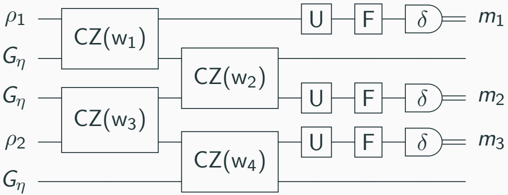

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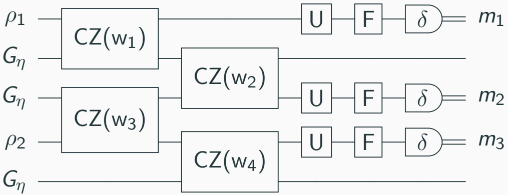

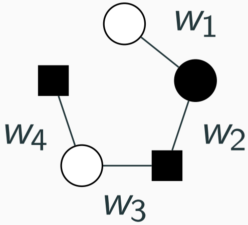

Graph states (cont.)

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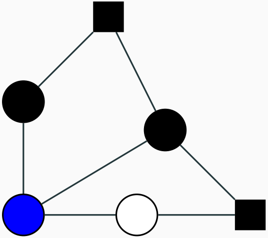

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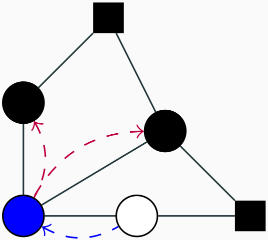

Causal flow

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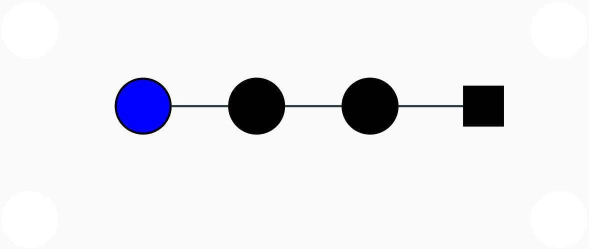

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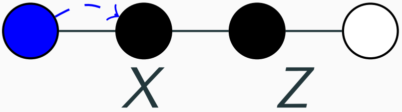

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- Corrections sum:

$$Z(a)Z(b) = Z(a+b)$$
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$$\mathsf{Z}(a)\mathsf{Z}(b) = \mathsf{Z}(a+b)$$
$$\mathsf{X}(a)\mathsf{X}(b) = \mathsf{X}(a+b)$$

- Linear equations:

$$A_i\vec{c} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{with} \quad \vec{c} \in \mathbb{R}^n$$

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- We obtain an explicit circuit extraction scheme for the MBQC protocol when the graph state has CV-flow.

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- If a graph has a CV-flow, and the same number of input nodes as outputs, then the MBQC procedure converges.
- CV-flow is essentially causal flow in the “wrong” basis.
- We obtain an explicit circuit extraction scheme for the MBQC protocol when the graph state has CV-flow.
- (Poly-time algorithm for finding a CV-flow if one exists.)

Some open questions

1. A necessity theorem?
2. Cleaner proofs with distributions?
3. Generalisation to other groups?
4. Counting flows and Blind Quantum Computation?