Flow Conditions for Continuous Variable Measurement-based Quantum Computing

Robert Booth, Damian Markham

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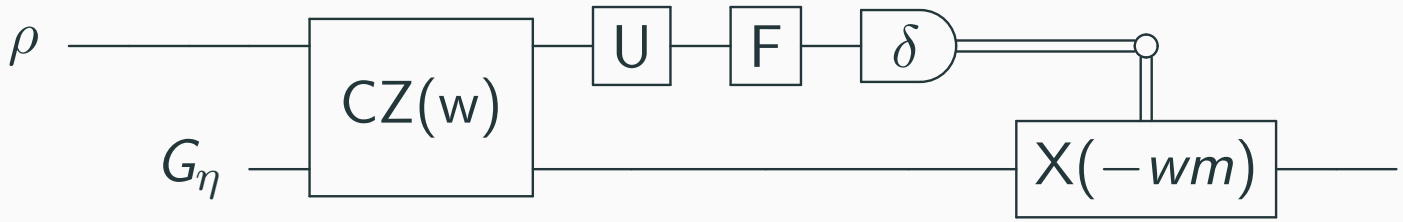
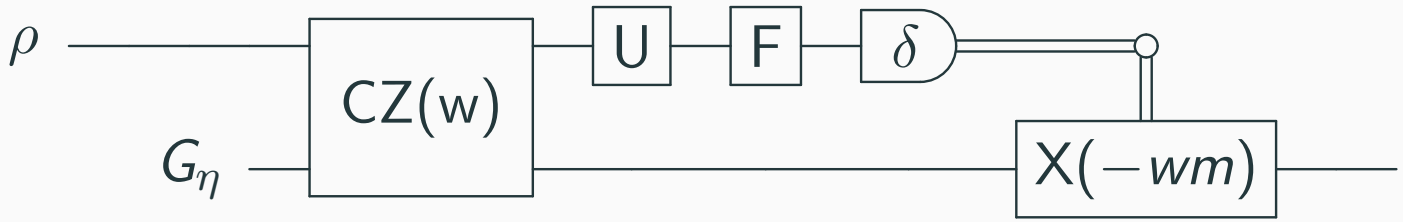
Measurement-based Quantum Computing



Measurement-based Quantum Computing

- Gate teleportation



Measurement-based Quantum Computing

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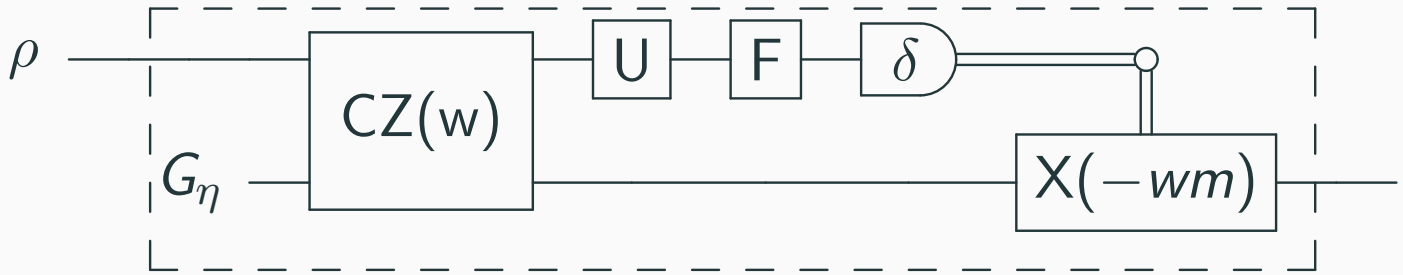
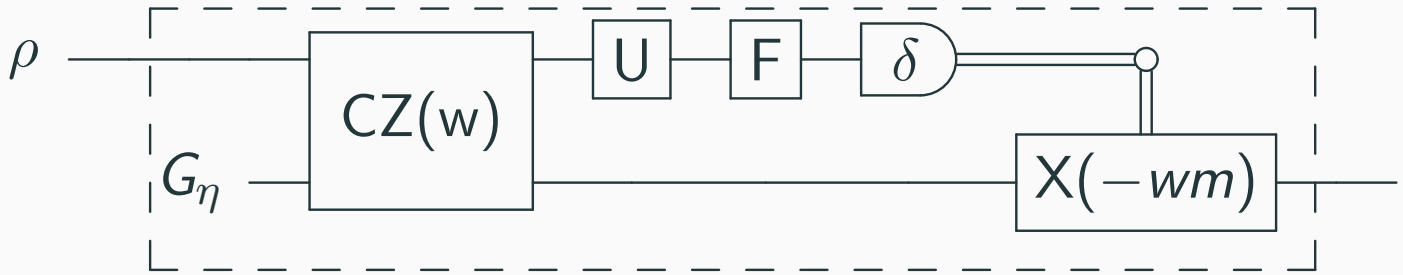


Measurement-based Quantum Computing

- Gate teleportation



- Main question: convergence?

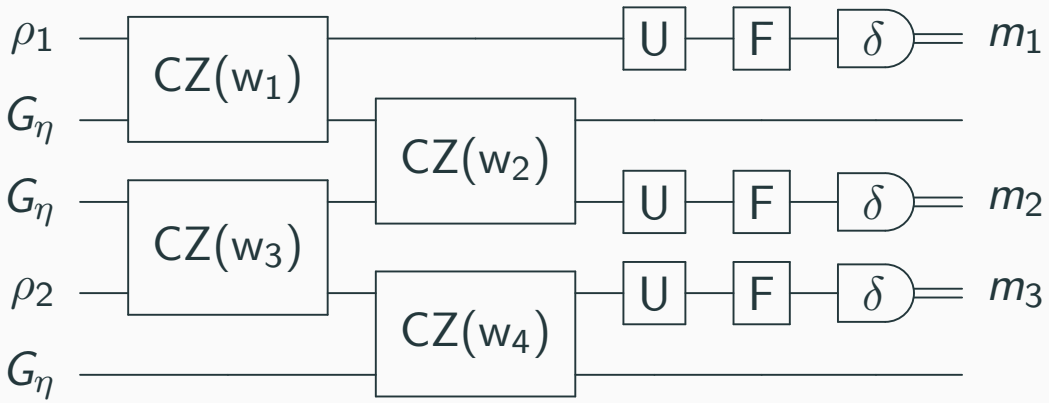
Measurement-based Quantum Computing (cont.)

Formally,

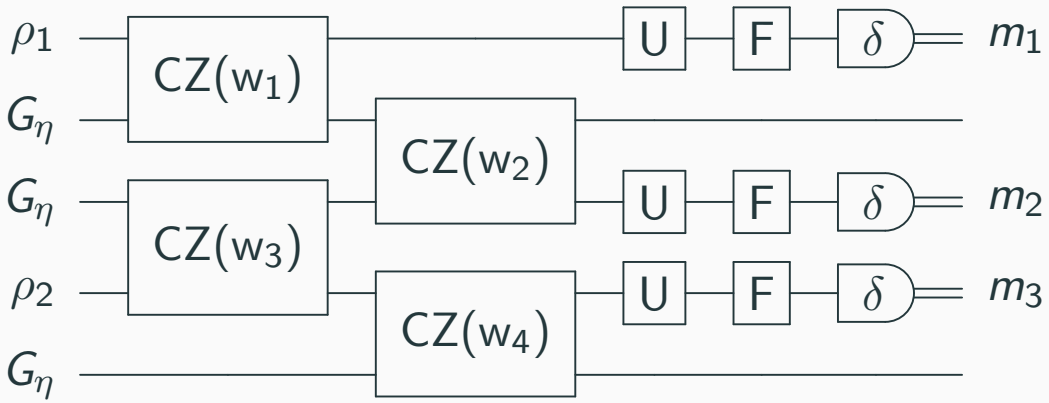
$$\lim_{\substack{\eta\to\infty \\ \delta\to 0}} \mathscr{C}_\eta^\delta[\rho] = \mathsf{S}(w)\mathsf{FU}[\rho]$$

MBQC protocols

MBQC protocols

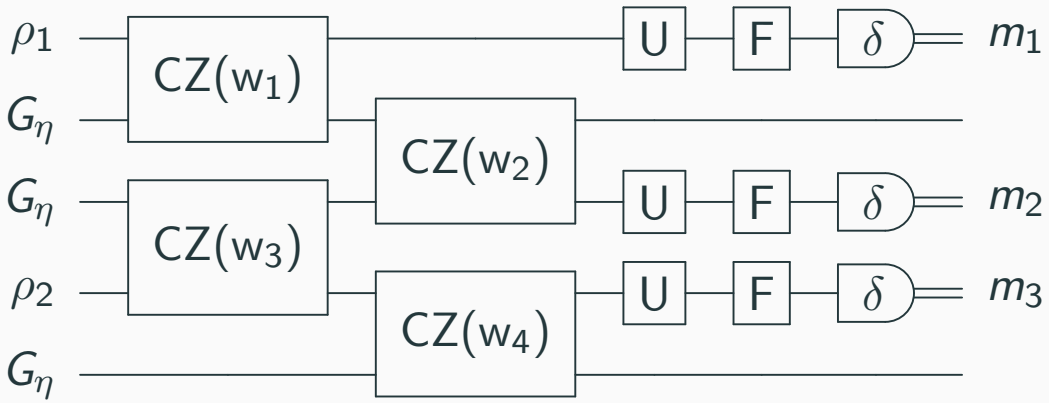


MBQC protocols



$$\lim_{\substack{\eta\to\infty \\ \delta\to 0}} \mathscr{M}_\eta^\delta[\rho_1 \otimes \rho_2]$$

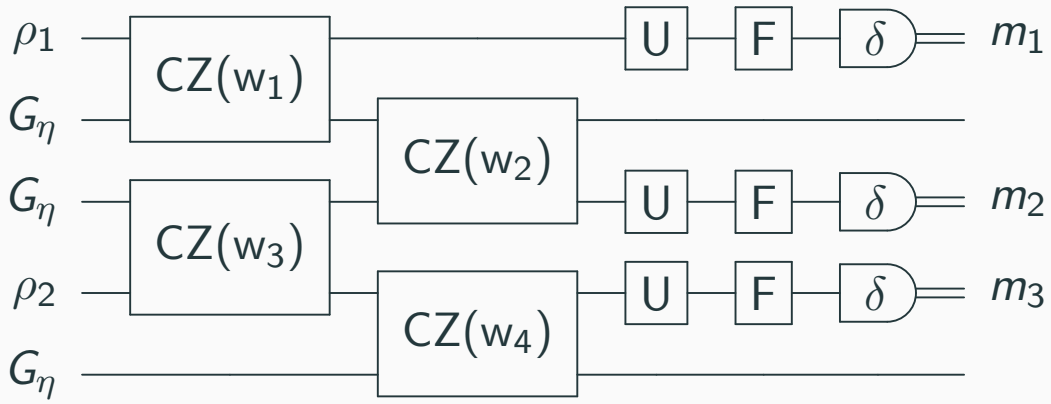
MBQC protocols



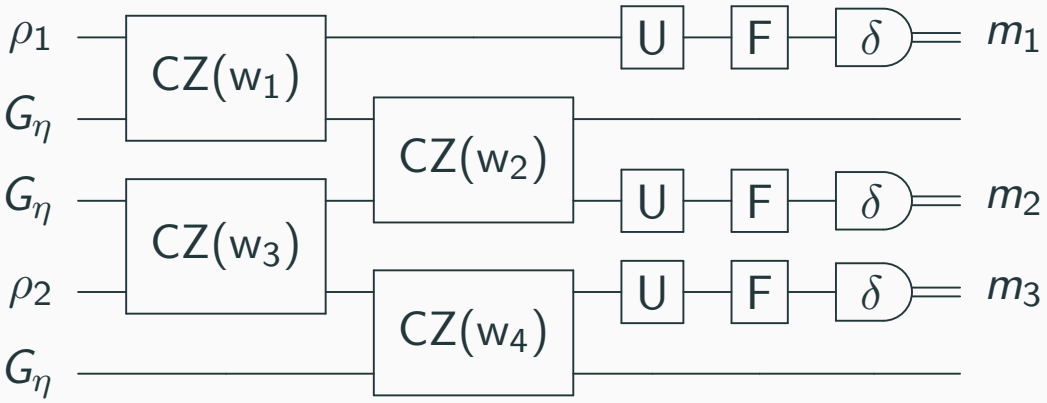
$$\lim_{\substack{\eta\to\infty \\ \delta\to 0}} \mathscr{M}_\eta^\delta[\rho_1 \otimes \rho_2]$$

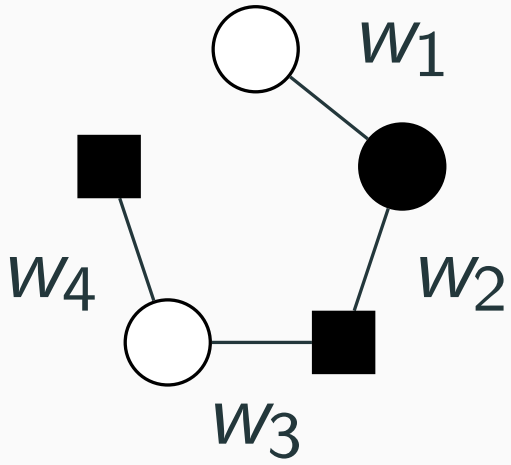
- Another question: runnability?

Graph states



Graph states





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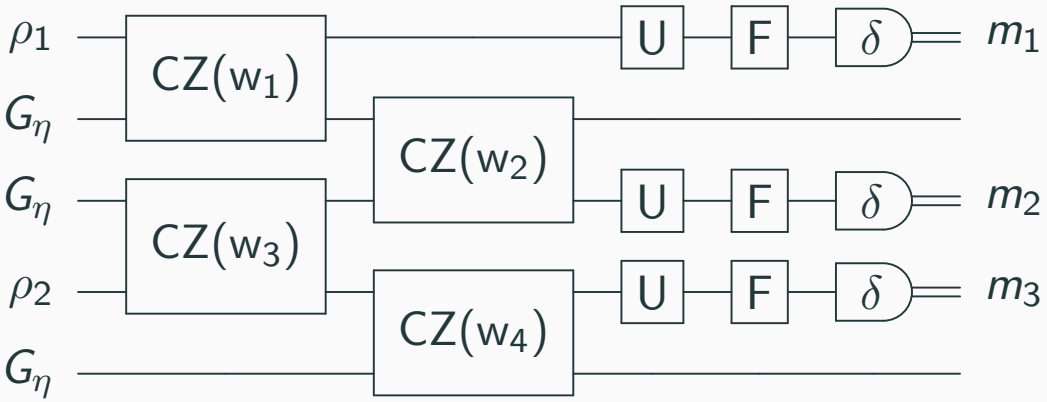
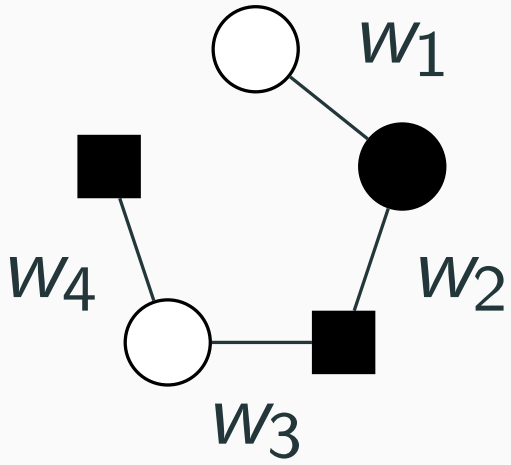




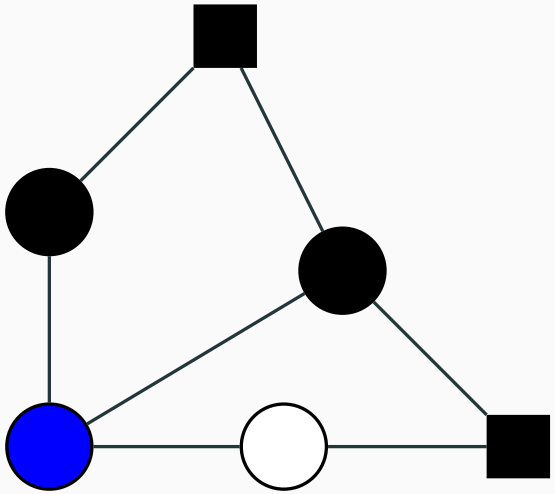
- Stabilisers: $K_i(s) = X_i(s) \prod_{j \in G(i)} Z_j(s)$

Graph states (cont.)

- Corrections?

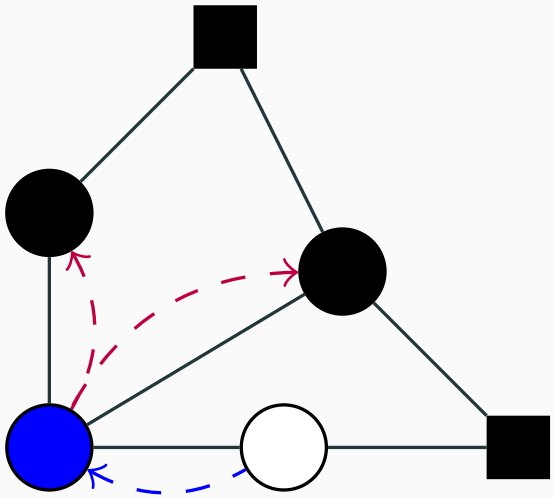
Graph states (cont.)

- Corrections?



Graph states (cont.)

- Corrections?



Causal flow

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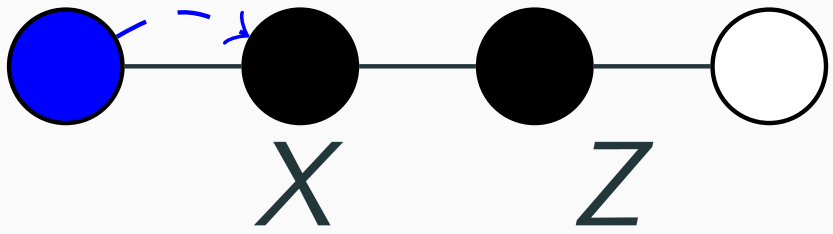
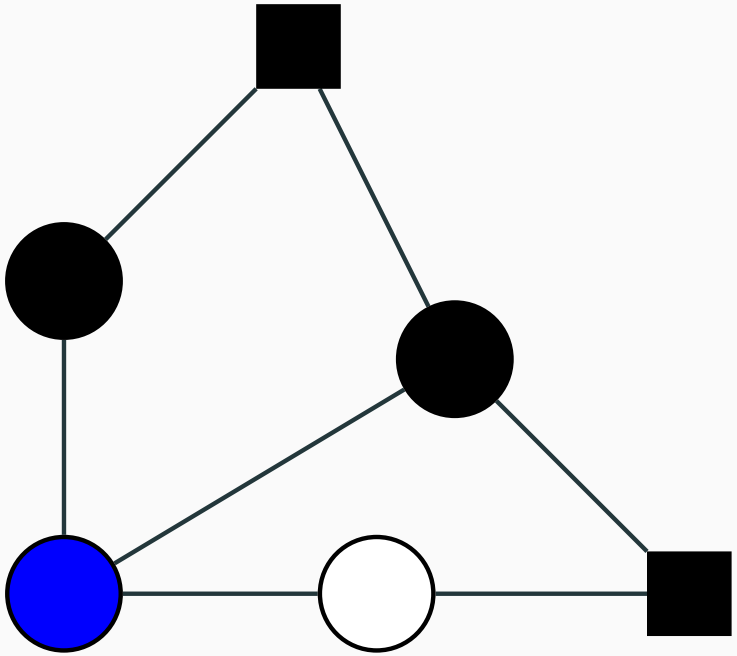


CV-flow



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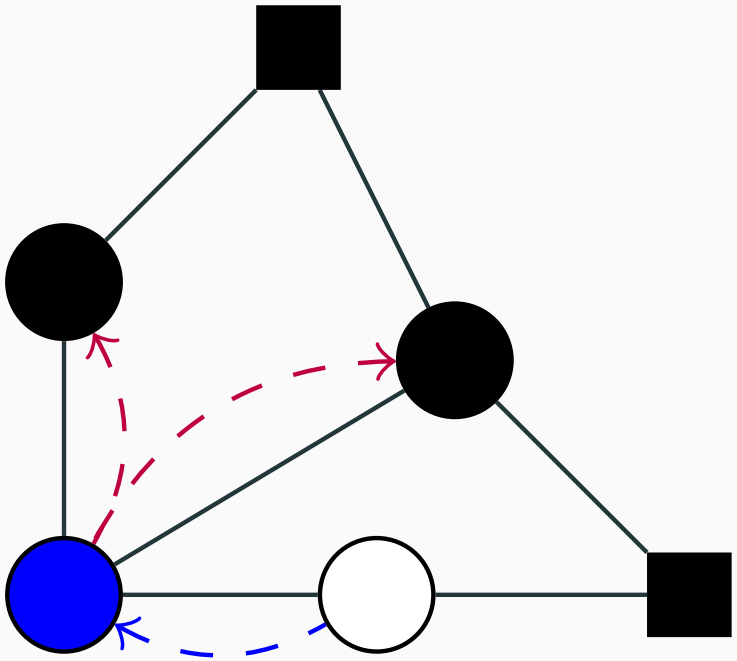
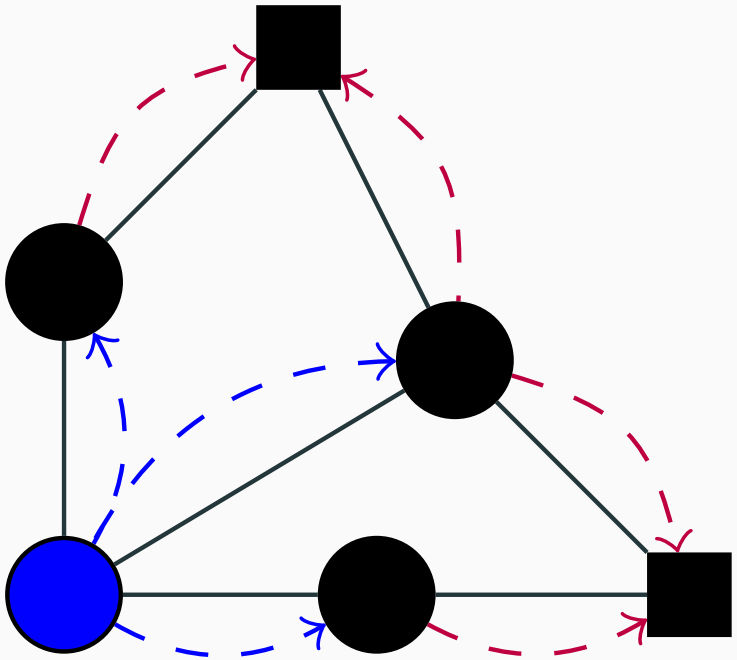
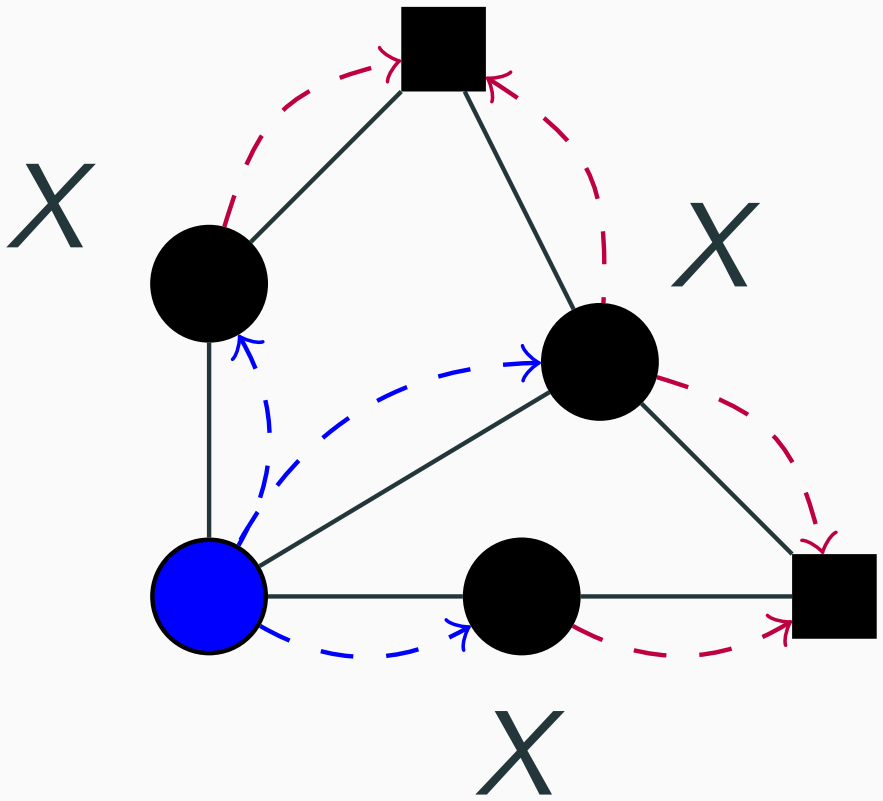


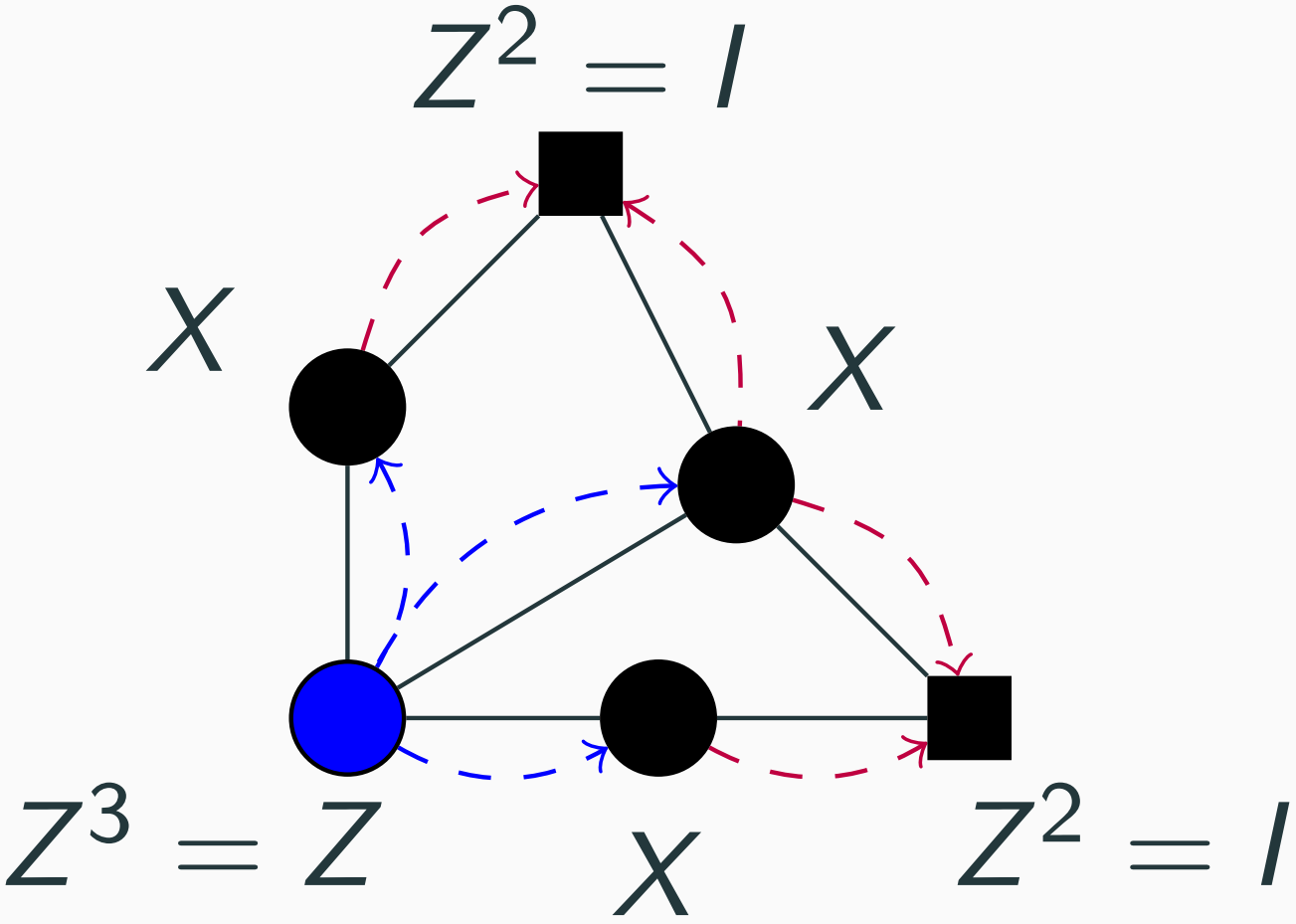
CV-flow



CV-flow



CV-flow



CV-flow (cont.)

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- Corrections sum:

$$\mathrm{Z}(a)\mathrm{Z}(b) = \mathrm{Z}(a+b)$$
$$\mathrm{X}(a)\mathrm{X}(b) = \mathrm{X}(a+b)$$

CV-flow (cont.)

- Corrections sum:

$$\mathrm{Z}(a)\mathrm{Z}(b) = \mathrm{Z}(a+b)$$
$$\mathrm{X}(a)\mathrm{X}(b) = \mathrm{X}(a+b)$$

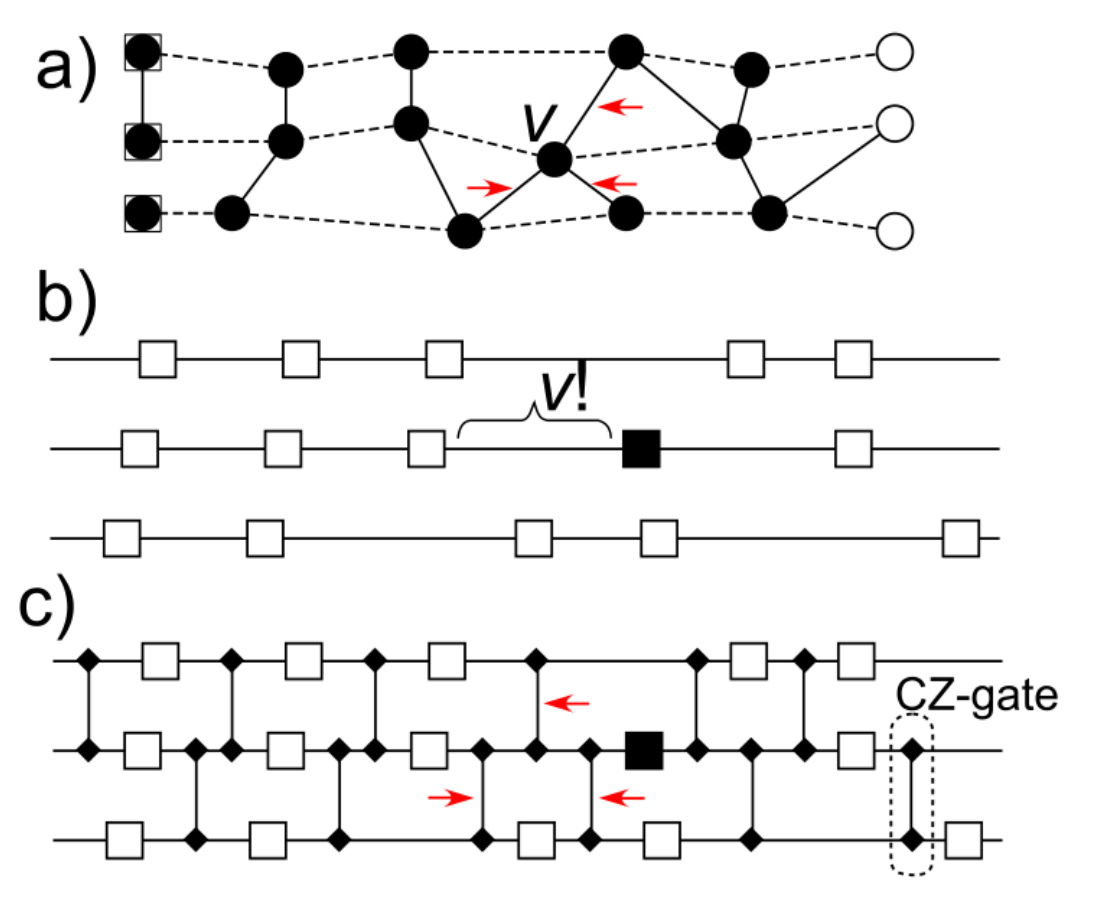
- Linear equations:

$$A_i \vec{c} = \begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix} \quad \text{with} \quad \vec{c} \in \mathbb{R}^n$$

Recap

Picture	Quantum Circuit	Graph State	Adjacency Matrices
Problem	Commute gates to make circuit runnable	Complete stabilisers	Solve linear equations

Convergence: causal flow



[1] Jisho Miyazaki, Michal Hajdušek, and Mio Murao. "An Analysis of the Trade-off between Spatial and Temporal Resources for Measurement-Based Quantum Computation". In: *Physical Review A* 91.5 (May 2015), p. 052302.

Takeaway

- If a graph has a CV-flow, and the same number of input nodes as outputs, then there is an MBQC procedure and it converges.
- CV-flow is essentially causal flow in the "wrong" basis.
- We obtain an explicit circuit extraction scheme for the MBQC protocol when the graph state has CV-flow.
- (Poly-time algorithm for finding a CV-flow if one exists.)