Flow Conditions for Continuous Variable Measurement-based Quantum Computing

Robert Booth, Damian Markham

June 5th, 2020


Disney
INTERACTIVE
STUDIOS

Causal flow and gflow

Causal flow and gflow

- Vincent Danos and Elham Kashefi. “Determinism in the One-Way Model”. In: *Physical Review A* 74.5 (Nov. 2006), p. 052310

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- D. E. Browne et al. "Generalized Flow and Determinism in Measurement-Based Quantum Computation". en. In: *New Journal of Physics* 9.8 (Aug. 2007), pp. 250–250
- Jisho Miyazaki, Michal Hajdušek, and Mio Murao. "An Analysis of the Trade-off between Spatial and Temporal Resources for Measurement-Based Quantum Computation". In: *Physical Review A* 91.5 (May 2015), p. 052302

Some motivation

- Nicolas C. Menicucci et al. "Universal Quantum Computation with Continuous-Variable Cluster States". In: *Physical Review Letters* 97.11 (Sept. 2006)
- Jun-ichi Yoshikawa et al. "Generation of One-Million-Mode Continuous-Variable Cluster State by Unlimited Time-Domain Multiplexing". In: *APL Photonics* 1.6 (Sept. 2016), p. 060801

Some motivation

- Anne Broadbent and Elham Kashefi. "Parallelizing Quantum Circuits". en. In: *Theoretical Computer Science* 410.26 (June 2009), pp. 2489–2510
- **duncan˙graph-theoretic˙2019**
- Atul Mantri et al. "Flow Ambiguity: A Path Towards Classically Driven Blind Quantum Computation". en. In: *Physical Review X* 7.3 (July 2017)

Continuous variables: states

- State space $\rightarrow$ Hilbert space:

$$f \in \mathrm{L}^2(\mathbb{R}, \mathbb{C}) \iff f : \mathbb{R} \rightarrow \mathbb{C} \quad \text{s.t.} \quad \int_{x \in \mathbb{R}} |f(x)|^2 < \infty$$

$$\mathrm{Prob}[\mathsf{Q} \in E \mid f] = \int_{x \in E} |f(x)|^2$$

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$$\mathrm{Prob}[\mathsf{Q} \in E \mid f] = \int_{x \in E} |f(x)|^2$$

- Countable orthonormal basis:

$$f = \sum_{n \in \mathbb{N}} c_n \xi_n \quad \text{where} \quad \xi_n \in \mathrm{L}^2(\mathbb{R}, \mathbb{C})$$

Continuous variables: transformations

- Gate set:

Seth Lloyd and Samuel L. Braunstein. "Quantum Computation over Continuous Variables". In: *Physical Review Letters* 82.8 (Feb. 1999), pp. 1784–1787.

Continuous variables: transformations

- Gate set:

$$\left\{\mathsf{F}, \exp(i\alpha\mathsf{Q}), \exp\left(i\beta\mathsf{Q}^2\right), \exp\left(i\gamma\mathsf{Q}^3\right), \exp(iw\mathsf{Q}\otimes\mathsf{Q})\right\}$$

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Continuous variables: transformations

- Gate set:

$$\left\{\mathsf{F}, \mathsf{U}(\alpha, \beta, \gamma), \mathsf{CZ}(w)\right\}$$

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Continuous variables: transformations

- Gate set:

$$\left\{\mathrm{F}, \mathrm{U}, \mathrm{CZ}(w)\right\}$$

- Gates used for correction in the MBQC procedure:

$$\mathrm{Z}(s) := \exp(-is\mathrm{P})$$
$$\mathrm{X}(s) := \exp(is\mathrm{Q})$$

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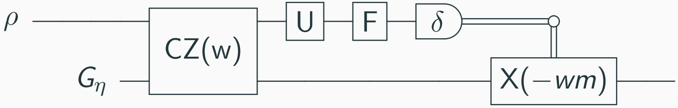
- $\Big($Lie theory $\rightsquigarrow$ $\exp : i\mathbb{R}[\mathsf{Q}, \mathsf{P}] \to \mathcal{U}(\mathcal{H})\Big)$

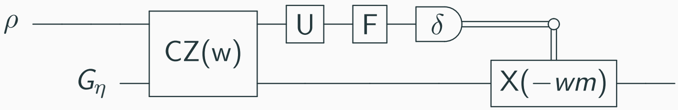
Seth Lloyd and Samuel L. Braunstein. "Quantum Computation over Continuous Variables". In: *Physical Review Letters* 82.8 (Feb. 1999), pp. 1784–1787.

Measurement-based Quantum Computing



Measurement-based Quantum Computing

- Gate teleportation



Measurement-based Quantum Computing

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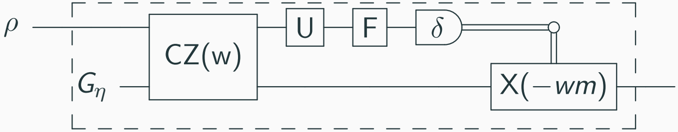
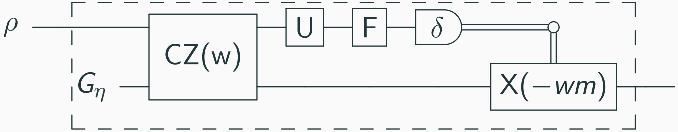


- Gate teleportation



$$\updownarrow$$

$$\rho \;—\boxed{\mathscr{C}_\eta^\delta}—\; \sim \mathrm{FU}[\rho]$$

- Problem 1: convergence?

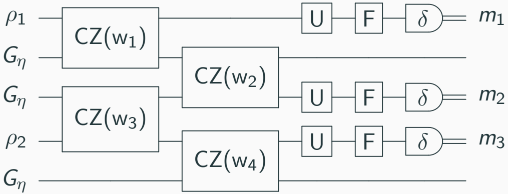
Measurement-based Quantum Computing (cont.)

Formally,

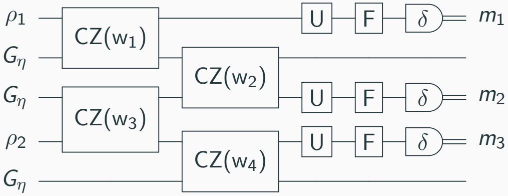
$$\lim_{\substack{\eta\to\infty \\ \delta\to 0}} \mathscr{C}_\eta^\delta[\rho] = \mathsf{S}(w)\mathsf{FU}[\rho]$$

MBQC protocols

MBQC protocols

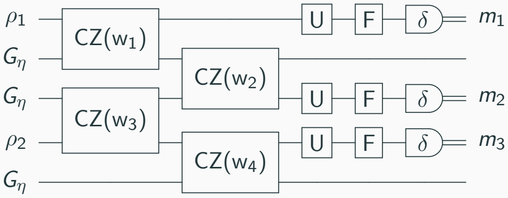


MBQC protocols



$$\lim_{\substack{\eta\to\infty \\ \delta\to 0}} \mathscr{M}_\eta^\delta[\rho_1 \otimes \rho_2]$$

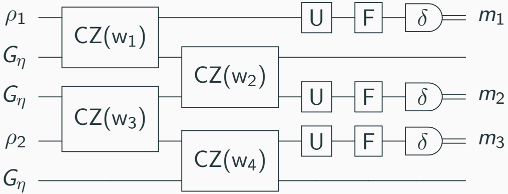
MBQC protocols



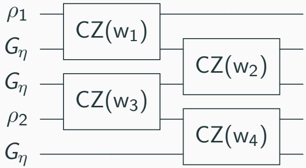
$$\lim_{\substack{\eta\to\infty\\ \delta\to 0}} \mathscr{M}_\eta^\delta[\rho_1 \otimes \rho_2]$$

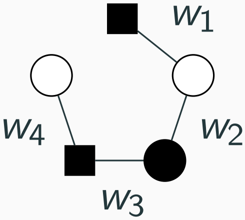
- Problem 1: convergence?
- Problem 2: runnability?

Graph states

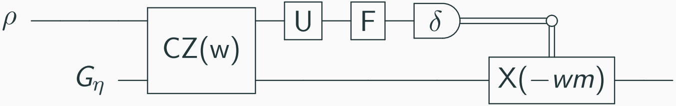


Graph states





Graph states (cont.)



$\updownarrow$

Problem I: Runnability

Causal flow

Causal flow

CV-flow: an example

Problem II: Convergence

Extraction: causal flow

CV-flow to causal flow

CV-flow: extraction

Convergence result

Formally, for any choice of input $\rho \in \mathcal{H}^{\otimes |O|}$ and any *local* unitaries from the gate set,

$$\lim_{\substack{\eta \to \infty \\ \delta \to 0}} \mathscr{M}_\eta^\delta[\rho] = \prod_{i \in \text{Layers}} \mathsf{T}^{(i)} \mathsf{U}^{(i)}_{\text{qausal}} \left[\rho \right]$$

Takeaway: results

- MBQC procedure based on CV-flow, which converges iff $|I| = |O|$.
- Explicit circuit extraction scheme for the MBQC protocol when the graph state has CV-flow.
- Poly-time algorithm for finding a CV-flow if one exists.
- Examples graphs that separate CV-flow from gflow

mhalla˙finding˙2008-1.

Some open questions

1. A (pseudo-)necessity theorem?
2. Several measurement planes?
3. Other rings?
4. Hypergraphs?

Thanks for watching!