# Measurement-based quantum computation beyond qubits and related problems

Robert I. Booth

EPiQC, October 26th 2021

# Related publications

- Robert I. Booth and Damian Markham. "Flow Conditions for Continuous Variable Measurement-Based Quantum Computing". Submitted to Quantum, Apr. 2021
- Robert I. Booth et al. "Outcome Determinism in Measurement-Based Quantum Computation with Qudits". Submitted to Quantum, Sept. 2021
- Robert I. Booth and Simon Perdrix. "Extracting Unitary Circuits from Measurement-Based Quantum Computations with Higher-Dimensional Systems". In preparation, 2021

# Causal flow and gflow


- Vincent Danos, Elham Kashefi, and Prakash Panangaden. "The Measurement Calculus". In: *Journal of the ACM* 54.2 (Apr. 2007), 8–es
- Vincent Danos and Elham Kashefi. "Determinism in the One-Way Model". In: *Physical Review A* 74.5 (Nov. 2006), p. 052310
- D. E. Browne et al. "Generalized Flow and Determinism in Measurement-Based Quantum Computation". In: *New Journal of Physics* 9.8 (Aug. 2007), pp. 250–250

# Some motivation

- Anne Broadbent and Elham Kashefi. “Parallelizing Quantum Circuits”. In: *Theoretical Computer Science* 410.26 (June 2009), pp. 2489–2510
- Ross Duncan et al. “Graph-Theoretic Simplification of Quantum Circuits with the ZX-Calculus”. In: *Quantum* 4 (June 2020), p. 279
- Atul Mantri et al. “Flow Ambiguity: A Path Towards Classically Driven Blind Quantum Computation”. In: *Physical Review X* 7.3 (July 2017)
- ...and many others!

# Quantum computing with qudits: states

- State space $\to$ $d$-dimensional Hilbert space:

$$\mathcal{H} = \mathrm{L}^2(\mathbb{Z}_d) \cong \mathbb{C}^d$$

$$|\psi\rangle \in \mathcal{H} \iff |\psi\rangle = \sum_{x \in \mathbb{Z}_d} c_x |x\rangle$$

- $d$ is prime $\implies$ $\mathbb{Z}_d$ is a field

## Quantum computing with qudits: gates

- Pauli gates

$$Z|x\rangle = e^{\frac{i2\pi x}{d}}|x\rangle = \omega^x|x\rangle \quad X|x\rangle = |x+1\rangle$$

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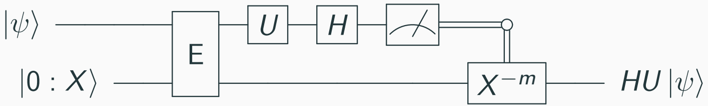
- These gates are not self-inverse

$$(X^a Z^b)^d = 1_{\mathcal{H}} \qquad H^4 = 1_{\mathcal{H}} \qquad E^d = 1_{\mathcal{H}\otimes\mathcal{H}}.$$

# Gate teleportation protocols



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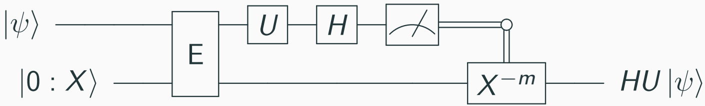


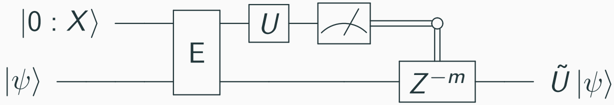

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**Lemma**

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**Lemma**

*$M, N \in \mathcal{M}(Q)$ if and only if there is $U \in SU(d)$ such that $M = UNU^\dagger$ and furthermore $[U, Q] = 0$.*

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- $M_u^{a,b}(\vec{\theta})$ : measurement of qudit $u$ in the measurement space $\mathcal{M}(X^a Z^b)$ with angles $\vec{\theta}$;

# Open graphs

$$\mathfrak{P} \simeq \left(\prod_{v \in O^c}^{\prec} X_{\mathbf{x}(v)}^{m_v} Z_{\mathbf{z}(v)}^{m_v} M_v^{a_v, b_v}(\vec{\theta}_v)\right) \left(\prod_{(u,v) \in G} E_{u,v}^{G_{uv}}\right) \left(\prod_{v \in I^c} N_v\right)$$

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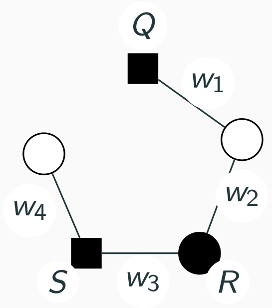


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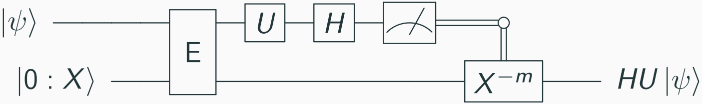
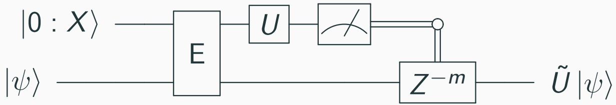
# Teleportation, graphically




# Rotation-gadget, graphically




# Outcome determinism

$$\mathfrak{P} : \rho \longmapsto \sum_{\vec{m} \in \mathbb{Z}_d^{O^c}} A_{\vec{m}}(\theta) \rho A_{\vec{m}}(\theta)^*$$

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- It is *strongly deterministic* if all the maps in its Kraus decomposition are equal up to a phase.
- It is robustly deterministic if it is uniformly and stepwise strongly deterministic.

# Open graphs

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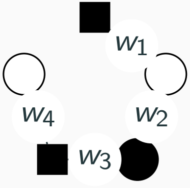


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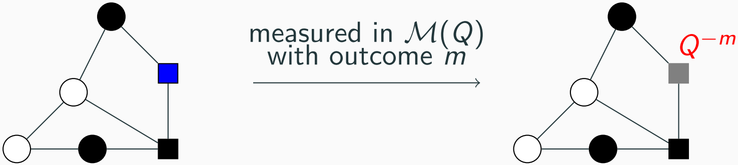
# Flow conditions, graphically

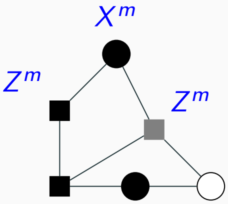
# Measurement errors




$$\langle m : M| = \langle 0 : M|\, Q^{-m}$$

# Corrections

1.

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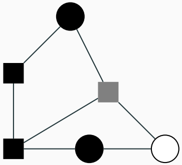

$$X^m \prod_{neighbours} Z^m \,|G\rangle = |G\rangle$$

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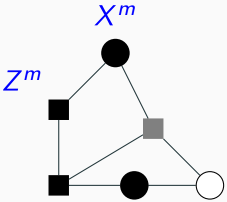


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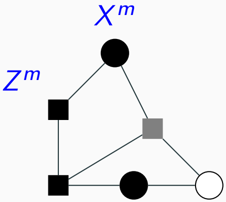

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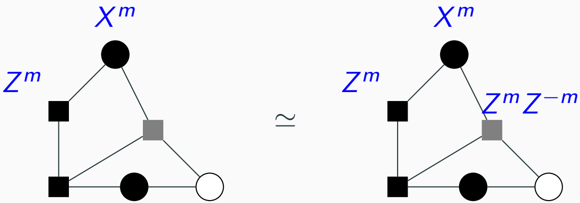
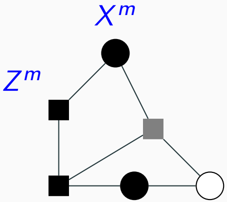
1. 

1. 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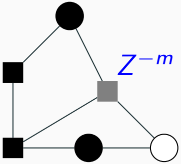

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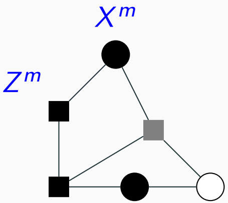
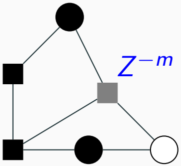
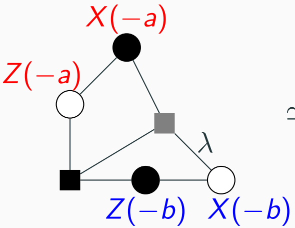
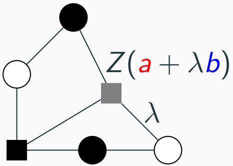
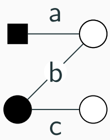
1.  $\simeq$ 



2.  $\simeq$ 

# $\mathbb{F}$-flow (simplified)

The open graph $G$ has an $\mathbb{F}$-flow if there is a lower triangular (square) matrix $C$ such that $A_G C$ is lower triangular.




$$\underbrace{\begin{pmatrix} 0 & 0 & a & 0 \\ 0 & 0 & b & c \\ a & b & 0 & 0 \\ 0 & c & 0 & 0 \end{pmatrix}}_{A_G}$$

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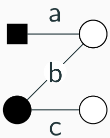


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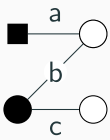


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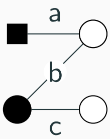


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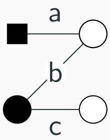


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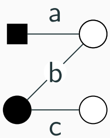


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# $\mathbb{F}$-flow (full)

**Definition**

$(G, I, O, \lambda)$ has an $\mathbb{F}$-flow if there exists $C \in \mathbb{F}^{V \times V}$ such that

1. $\forall u \in O^c$, $\lambda(u) = (C_{uu}, (GC)_{uu})$;
2. $C_{uv} = 0$ whenever $u \in I$ or $v \in O$;
3. $C$ and $GC$ are lower triangular.

# Main results: necessary and sufficient

**Theorem**

*Suppose an open graph has $\mathbb{Z}_d$-flow, then the corresponding MBQC is runnable and robustly deterministic. Furthermore, for a given choice of measurements, it realises an isometry $\mathcal{H}^{\otimes I} \to \mathcal{H}^{\otimes O}$.*

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**Theorem**

*If an MBQC is runnable and robustly deterministic, then the underlying open graph has a $\mathbb{Z}_d$-flow.*

**Theorem**

*There is a poly-time algorithm that finds an optimal $\mathbb{Z}_d$-flow for an open graph whenever one exists.*

# Proof of converse

$$|G(\phi)\rangle \longmapsto X^m_{\mathbf{x}(u)} Z^m_{\mathbf{z}(u)} \left( \langle m : \mathbf{M}(u)| \bigotimes_{v \in O^c \setminus \{u\}} \langle 0 : \mathbf{M}(v)| \right) |G(\phi)\rangle$$

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= & X^{m}_{\mathbf{x}(u)} Z^{m}_{\mathbf{z}(u)} \left( \bigotimes_{v \in O^{c}} \langle 0 : \mathbf{M}(v)| \right) Q^{m}_{u} |G(\phi)\rangle ,
\end{aligned}
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&= \left( \bigotimes_{v \in O^{\mathrm{c}}} \langle 0 : \mathbf{M}(v)| \right) X^m_{\mathbf{x}(u)} Z^m_{\mathbf{z}(u)} Q^m_u \, |G(\phi)\rangle
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|G(\phi)\rangle \longmapsto & X^m_{\mathbf{x}(u)} Z^m_{\mathbf{z}(u)} \left( \langle m : \mathbf{M}(u)| \bigotimes_{v \in O^{\mathsf{c}} \setminus \{u\}} \langle 0 : \mathbf{M}(v)| \right) |G(\phi)\rangle \\
&= X^m_{\mathbf{x}(u)} Z^m_{\mathbf{z}(u)} \left( \bigotimes_{v \in O^{\mathsf{c}}} \langle 0 : \mathbf{M}(v)| \right) Q^m_u |G(\phi)\rangle , \\
&= \left( \bigotimes_{v \in O^{\mathsf{c}}} \langle 0 : \mathbf{M}(v)| \right) X^m_{\mathbf{x}(u)} Z^m_{\mathbf{z}(u)} Q^m_u |G(\phi)\rangle \\
&= \bigotimes_{v \in O^{\mathsf{c}}} \langle 0 : \mathbf{M}(v)|G(\phi)\rangle .
\end{aligned}$$

# Proof of converse (cont.)

$$\left(\bigotimes_{v\in O^{\mathsf{c}}} \langle 0:\mathbf{M}(v)|\right) X^m_{\mathbf{x}(u)} Z^m_{\mathbf{z}(u)} Q^m_u \,|G(\phi)\rangle = \bigotimes_{v\in O^{\mathsf{c}}} \langle 0:\mathbf{M}(v)|G(\phi)\rangle\,.$$

$$\Downarrow$$

$$X^m_{\mathbf{x}(u)} Z^m_{\mathbf{z}(u)} Q^m_u \,|G(\phi)\rangle = |G(\phi)\rangle\,.$$

# Circuit extraction

# Difficulties

- we know how to extract gate teleportations:

$$\blacksquare\!\!-\!\!\!-\!\!\!-\!\!\bigcirc = -\boxed{U}-\boxed{H}- = -\boxed{J}- \tag{1}$$

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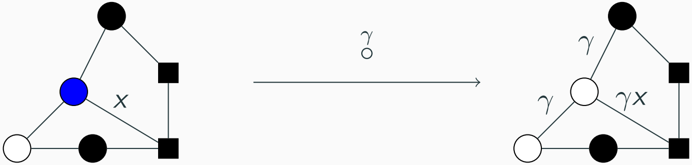
The solution: local Clifford operations

# Local scaling




$$G \overset{\gamma}{\circ} w = D_\gamma^w G D_\gamma^w \quad \text{and} \quad C^{\overset{\gamma}{\circ} w} = D_{\gamma^{-1}}^w C$$

# Local complementation

$$G \overset{\gamma}{\star} w := G + \gamma G 1_{\{w\}} 1_{\{w\}}^{\mathsf{T}} G - \gamma D^2_{Ge_w}.$$

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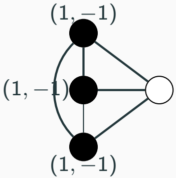
- they preserve the semantics of the pattern (up to updating the choice of measurements);
- they act simultaneously on both the connectivity of the open graph and its labelling;
- this action depends on the connectivity itself...

# The extraction algorithm (informally)

1. Using local Cliffords, we can reduce the graph, so that all labels are either $(0, 1)$ or $(1, 0)$; i.e. measurements are $\mathcal{M}(Z)$ or $\mathcal{M}(X)$;
2. any vertices connected to the outputs and measured in $\mathcal{M}(Z)$ can be extracted using teleportation;
3. if the only vertices connected to outputs are measured in $\mathcal{M}(X)$, then there is a special procedure to map them to $\mathcal{M}(Z)$.

# Circuit extraction: an example



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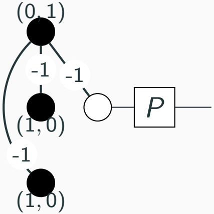

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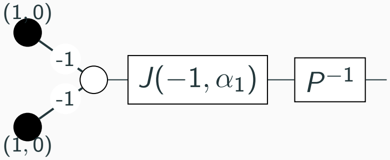

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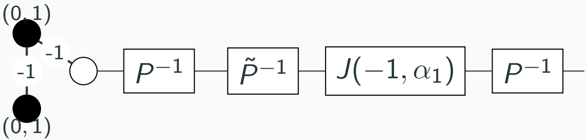

# Circuit extraction: an example

$$
-\boxed{J(-1,\alpha_3)}-\boxed{J(-1,\alpha_2)}-\boxed{P^{-1}}-\boxed{\tilde{P}^{-1}}-\boxed{J(-1,\alpha_1)}-\boxed{P^{-1}}-
$$

# Future work

1. improvements to the extraction algorithm;
2. Pauli $\mathbb{F}$-flow?
3. extraction of qudit ZX-calculus diagrams

Thanks for listening!

# A technical lemma

**Lemma**

*Let* $|\phi\rangle, |\phi'\rangle$ *be two states of a register* $V$ *of qudits,* $Q$ *a Pauli operator and fix some* $v \in V$. *If for every measurement* $M \in \mathcal{M}(Q_u)$ *of the qudit* $u$ *and every* $m \in \mathbb{Z}_d$, *we have*

$$\langle m : M \mid \phi\rangle \simeq \langle m : M \mid \phi'\rangle \quad \text{and} \quad \|\langle m : M \mid \phi\rangle\| = \frac{1}{\sqrt{d}} = \|\langle m : M \mid \phi'\rangle\|, \tag{2}$$

*then at least one of the following holds:*

1. $|\phi\rangle \simeq |\phi'\rangle$;
2. $|\phi\rangle$ *and* $|\phi'\rangle$ *are separable and there are* $x, y \in \mathbb{Z}_d$ *and* $|\psi\rangle \in \mathcal{H}_{V\setminus\{v\}}$ *such that*

$$|\phi\rangle = |x : Q\rangle_v \otimes |\psi\rangle \quad \text{and} \quad \left|\phi'\right\rangle \simeq |y : Q\rangle_v \otimes |\psi\rangle, \tag{3}$$

*where* $|x : Q\rangle$ *is the eigenvector of* $Q$ *associated with eigenvalue* $\omega^x$.