A symplectic vision for the ZX-calculus

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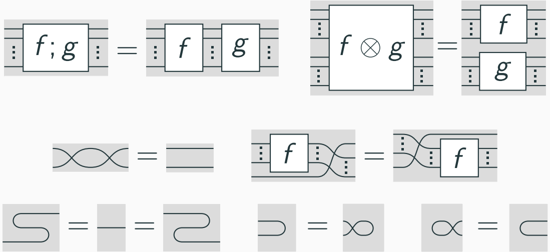
May 3, 2024

String diagrams

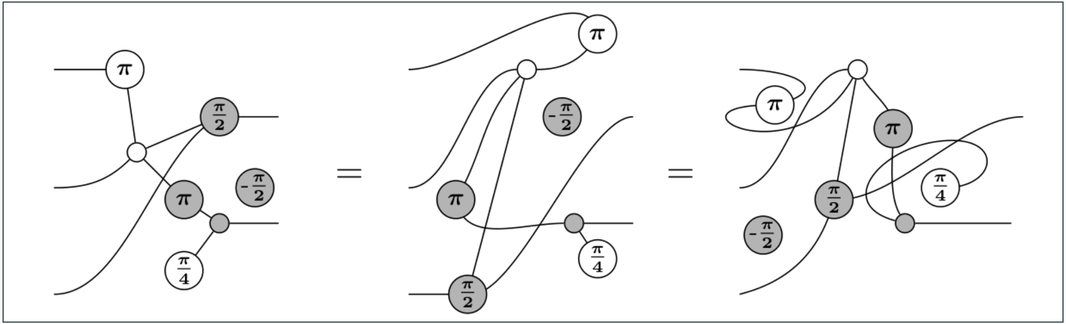
Compact-closed props admit string diagrams with the syntax:



We can turn any compact-closed category into a prop:

It's an ongoing area of research to find generators and equations for compact-closed categories in terms of string diagrams.

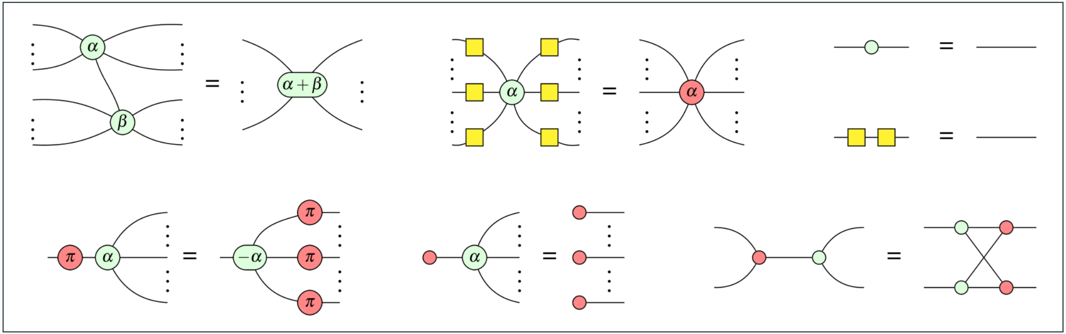
The ZX-calculus



Useful for: optimisation, classical simulation, and much more... see the website, `zxcalculus.com` !

John van de Wetering. **ZX-calculus for the working quantum computer scientist.** Dec. 27, 2020.

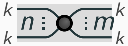
ZX-calculus equations

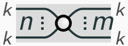


Thomas McElvanney and Miriam Backens. **"Complete flow-preserving rewrite rules for MBQC patterns with Pauli measurements"**. In: *Proceedings 19th International Conference on Quantum Physics and Logic*. Quantum Physics and Logic (QPL 2022). June 1, 2022.

Graphical linear algebra: generators

$\mathrm{GLA}_{\mathbb{K}}$ is generated by:



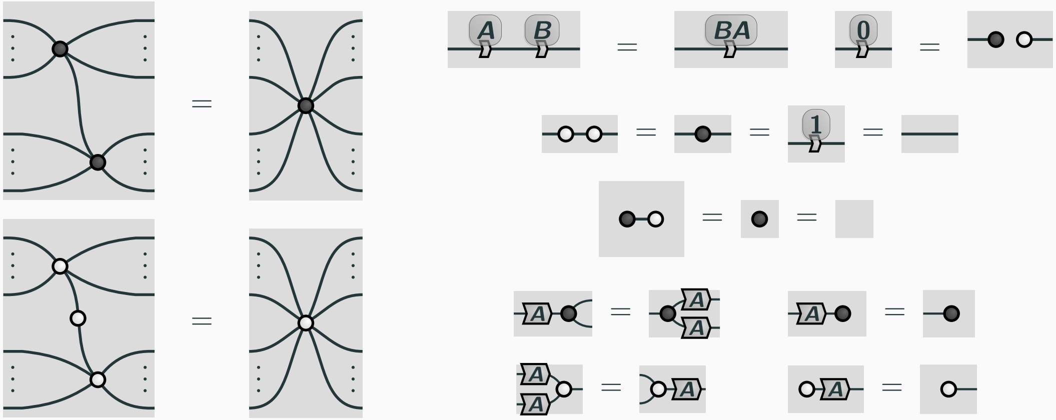


$= \mathrm{im}(A)$

$= \ker(A)$

Fabio Zanasi. **"Interacting Hopf Algebras: the theory of linear systems".** PhD thesis. Ecole Normale Superieure de Lyon, May 4, 2018.

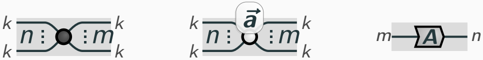
Graphical linear algebra: equations

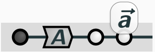
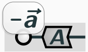


Theorem. $\mathrm{GLA}_{\mathbb{K}}$ is equivalent to the category of $\mathbb{K}$-linear relations.

Graphical affine algebra: generators

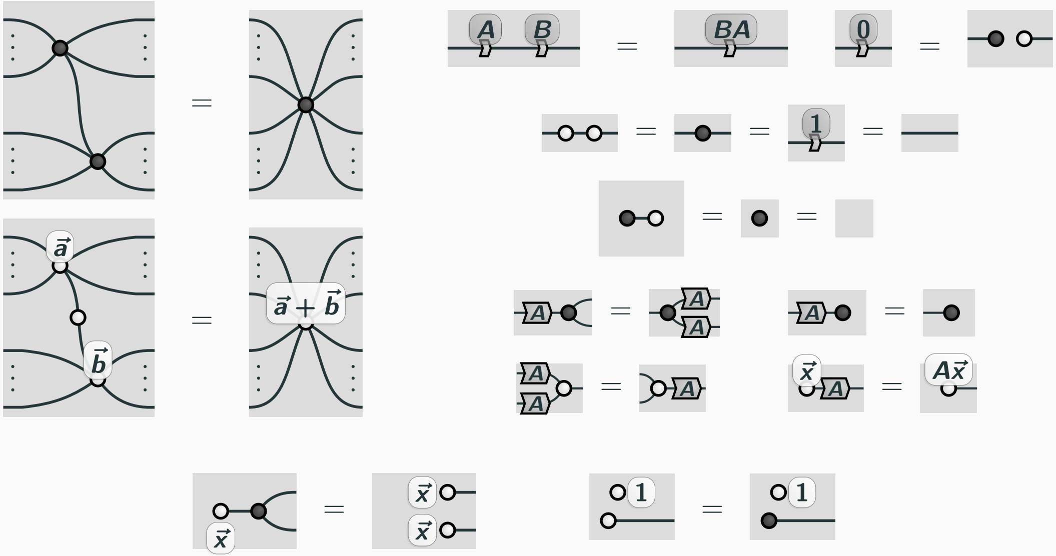
$\mathsf{GAA}_{\mathbb{K}}$ is generated by:



 $= \mathrm{im}(A + \vec{a})$  $= \ker(A + \vec{a})$

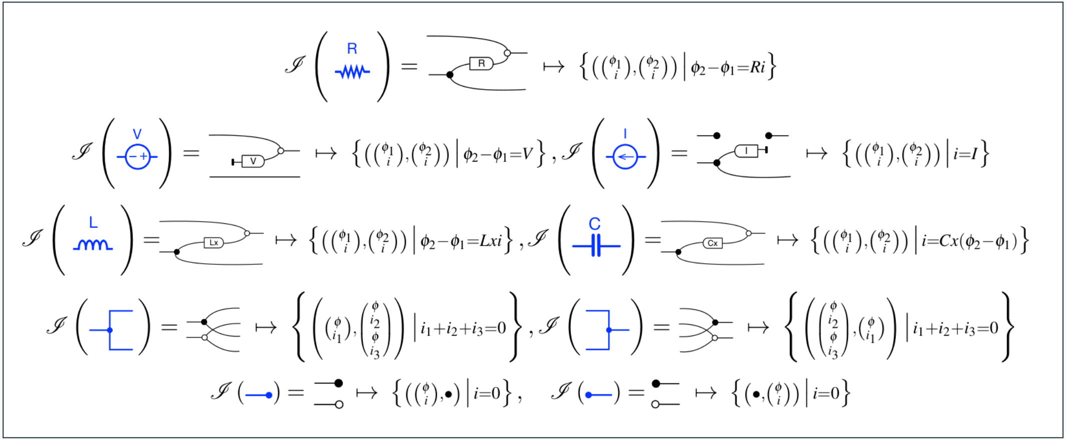
Filippo Bonchi et al. **"Graphical Affine Algebra".** In: *2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS)*. 2019 34th Annual ACM/IEEE Symposium on Logic in Computer Science (LICS). Vancouver, BC, Canada: IEEE, June 2019, pp. 1–12.

Graphical affine algebra: equations



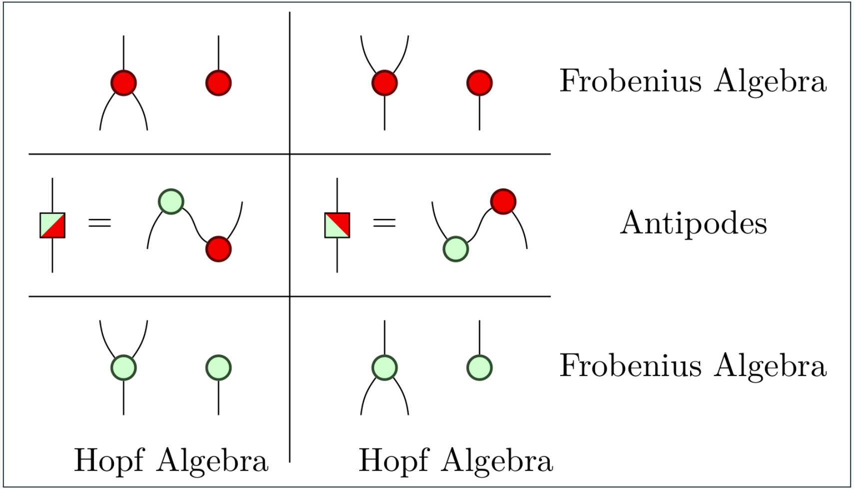
Theorem. $\mathrm{GAA}_{\mathbb{K}}$ is equivalent to the category of $\mathbb{K}$-affine relations.

Some applications



Guillaume Boisseau and Paweł Sobociński. **"String Diagrammatic Electrical Circuit Theory".** In: *Electronic Proceedings in Theoretical Computer Science*. Fourth International Conference on Applied Category Theory (ACT 2021). Vol. 372. Nov. 3, 2022, pp. 178–191.

What connection is there between these research programs?



Joseph Collins and Ross Duncan. **"Hopf-Frobenius Algebras and a Simpler Drinfeld Double".** In: *Electronic Proceedings in Theoretical Computer Science* 318 (May 1, 2020), pp. 150–180.

Black boxing

$$
\begin{tikzcd}
\widetilde{\mathrm{Circ}} \arrow[r, "P"] \arrow[d, hook] & \mathrm{Circ} \arrow[r, "G"] \arrow[d, hook] & \mathrm{FinCospan} \arrow[r, "H"] & \mathrm{FinCorel} \arrow[d, "K"] \\
\widetilde{\mathrm{Circ}}_k \arrow[r, "P"] \arrow[d, "T"'] & \mathrm{Circ}_k \arrow[rr, "\blacksquare"] & & \mathrm{LagRel}_k \arrow[d, hook] \\
\mathrm{SigFlow}_k \arrow[rrr, "\square"] & & & \mathrm{FinRel}_k .
\end{tikzcd}
$$

John C. Baez, Brandon Coya, and Franciscus Rebro. **"Props in Network Theory".** In: *Theory and Application of Categories* 33.25 (June 1, 2018), pp. 727–783.

Lagrangian relations

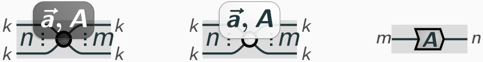
A linear subspace $W \subseteq \mathbb{K}^{2n}$ is Lagrangian if:

A linear relation $R : \mathbb{K}^{2m} \to \mathbb{K}^{2n}$ is Lagrangian if:

Cole Comfort and Aleks Kissinger. **"A Graphical Calculus for Lagrangian Relations".** In: *arXiv.org*. Applied Category Theory 2021. Vol. 372. EPTCS, May 13, 2021, pp. 338–351.

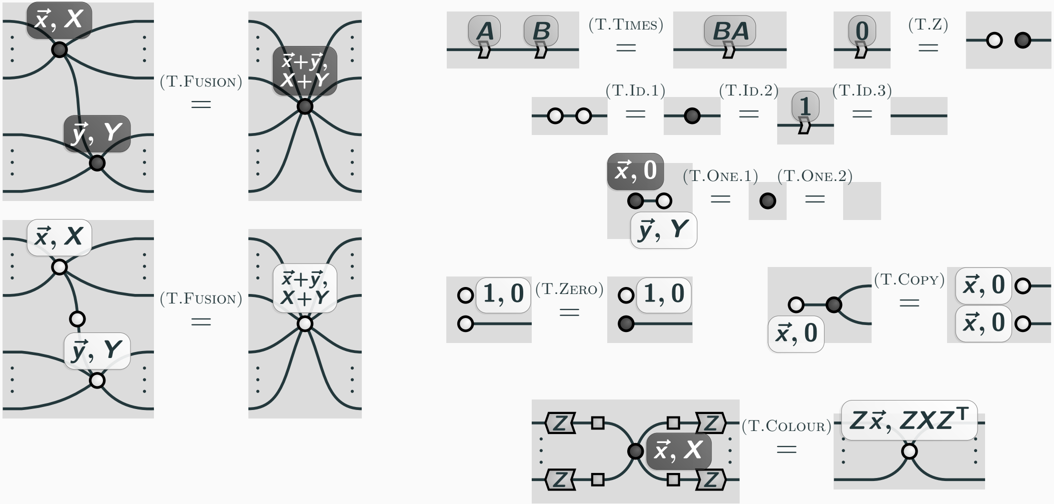
Graphical symplectic algebra: generators

$\mathrm{GSA}_{\mathbb{K}}$ is generated by:



Robert I. Booth, Titouan Carette, and Cole Comfort. **Graphical Symplectic Algebra.** Jan. 30, 2024.

Graphical symplectic algebra: equations



Theorem. $\mathrm{GSA}_{\mathbb{K}}$ is equivalent to the category of $\mathbb{K}$-affine Lagrangian relations.

Back to the ZX-calculus

Theorem. There is an equivalence* $\mathsf{AffLagRel}_{\mathbb{F}_p} \simeq \mathsf{ZX}_p^{\mathrm{Stab}} \subseteq \mathsf{Hilb}_p$.

Coisotropic relations

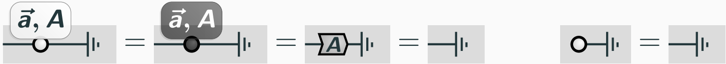
A linear subspace $W \subseteq \mathbb{K}^{2n}$ is coisotropic if:

A linear relation $R : \mathbb{K}^{2m} \to \mathbb{K}^{2n}$ is coisotropic if:

(**Theorem.** $CPM(\mathsf{AffLagRel}_{\mathbb{K}}) \simeq \mathsf{AffCoIsotRel}_{\mathbb{K}}$.)

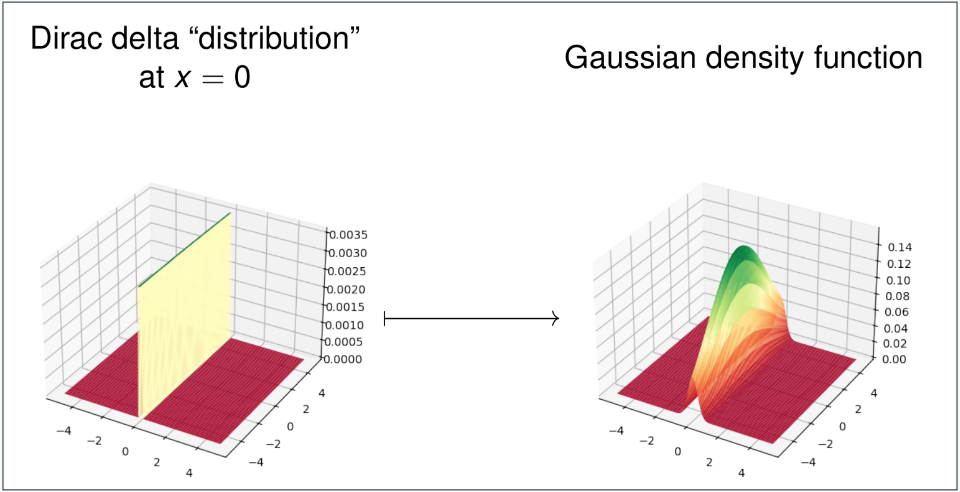
Adding discarding

The extension of $\mathrm{GSA}_{\mathbb{K}}$ by  such that



is complete for $\mathrm{AffCoIsotRel}_{\mathbb{K}}$.

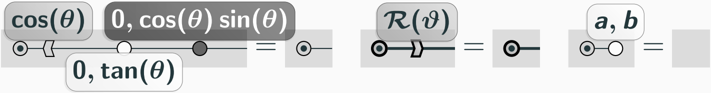
Gaussians



(**Theorem.** $\mathit{CPM}(\mathrm{AffLagRel}_{\mathbb{K}}) \simeq \mathrm{QGaussRel}$.)

Gaussians are also a discard

The extension of $\mathrm{GSA}_{\mathbb{R}}$ by ⊙– such that



is complete for QGaussRel.

The end?

The CPM construction