A graphical language for Gaussian quantum processes

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The paper

Complete equational theories for classical and quantum Gaussian relations

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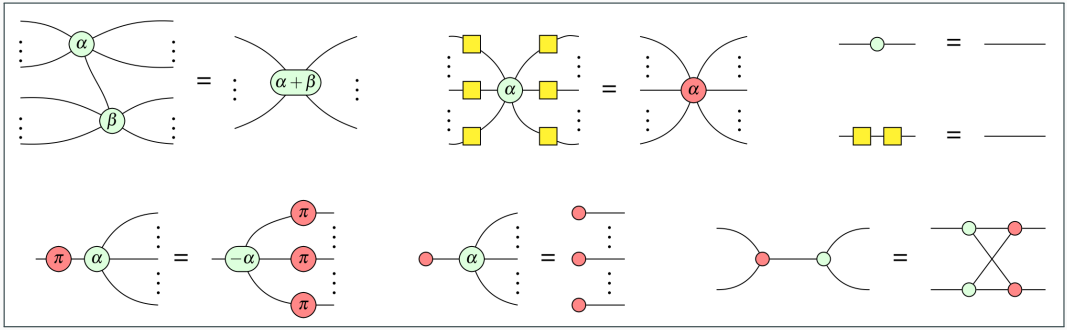
Abstract

We give generators and relations for the hypergraph props of Gaussian relations [65] and positive affine Lagrangian relations. The former extends Gaussian probabilistic processes by completely-uninformative priors, and the latter extends Gaussian quantum mechanics with infinitely-squeezed states. These presentations are given by adding a generator to the presentation of real affine relations and of real affine Lagrangian relations which freely codiscards effects, as well as certain rotations.

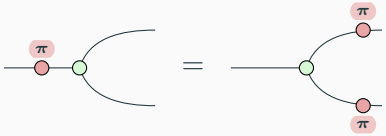
The presentation of positive affine Lagrangian relations provides a rigorous justification for many common yet informal calculations in the quantum physics literature involving infinite-squeezing. Our presentation naturally extends Menicucci et al.'s graph-theoretic representation of Gaussian quantum states [59] with a representation for Gaussian transformations. We interpret the LOv-calculus [27], a diagrammatic calculus for reasoning about passive linear-optical quantum circuits in our presentation of positive affine Lagrangian relations. Moreover, we show how our presentation allows for additional optical operations such as active squeezing.

The stabiliser ZX-calculus



Tommy McElvanney and Miriam Backens. **"Complete Flow-Preserving Rewrite Rules for MBQC Patterns with Pauli Measurements".** In: *QPL 2022* (May 4, 2022).

Stabilisers in the stabiliser ZX-calculus



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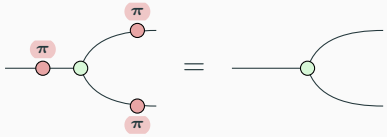

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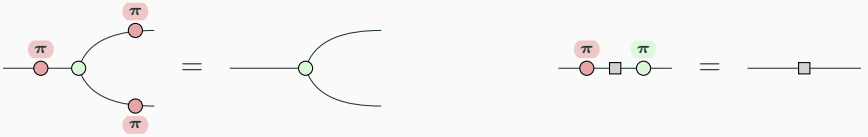

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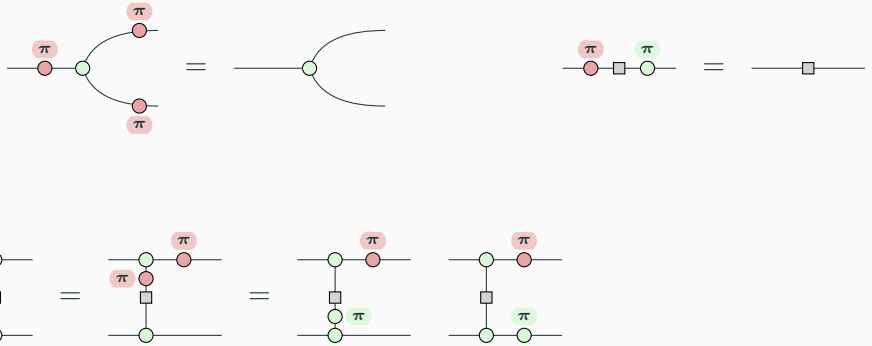

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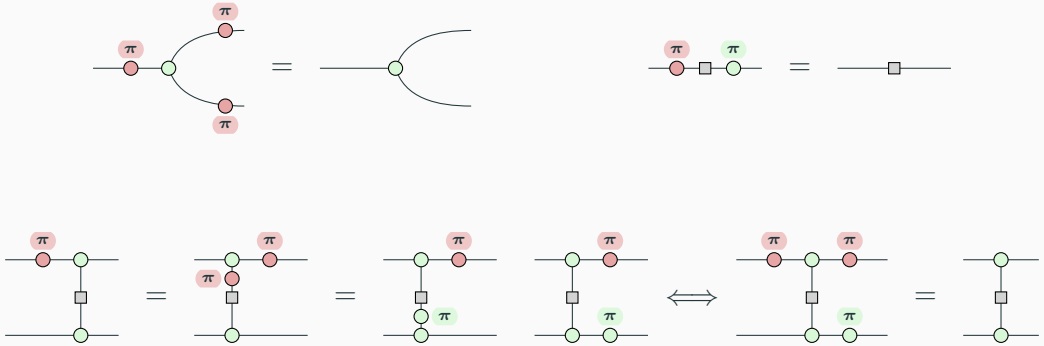
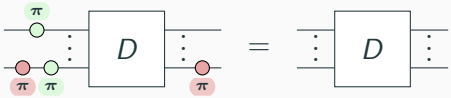


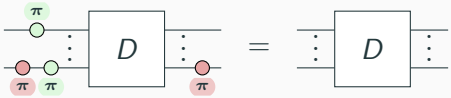
Stabilisers in the stabiliser ZX-calculus

Stabilisers of a diagram:



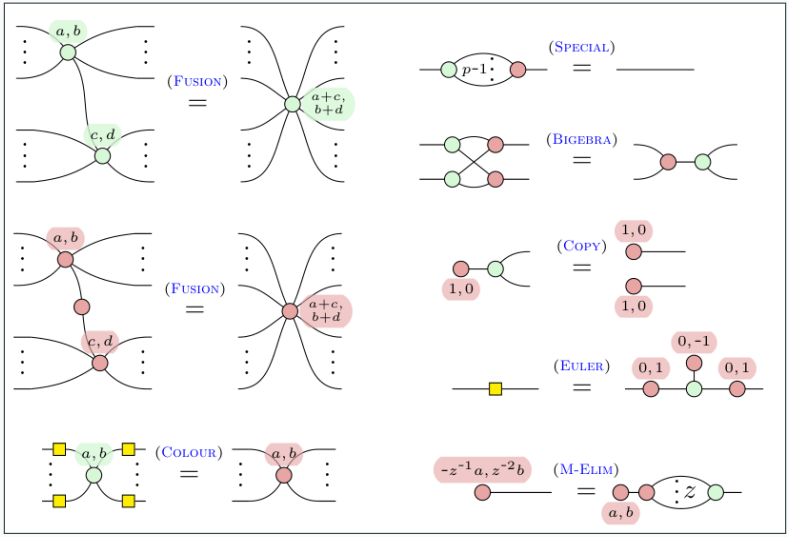
Stabilisers in the stabiliser ZX-calculus

Stabilisers of a diagram:



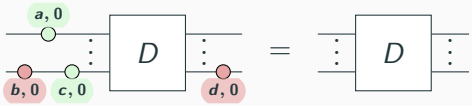
The collection of all such stabilisers of D gives an alternative semantics to the usual Hilbert space one!

The stabiliser ZX-calculus in prime dimensions

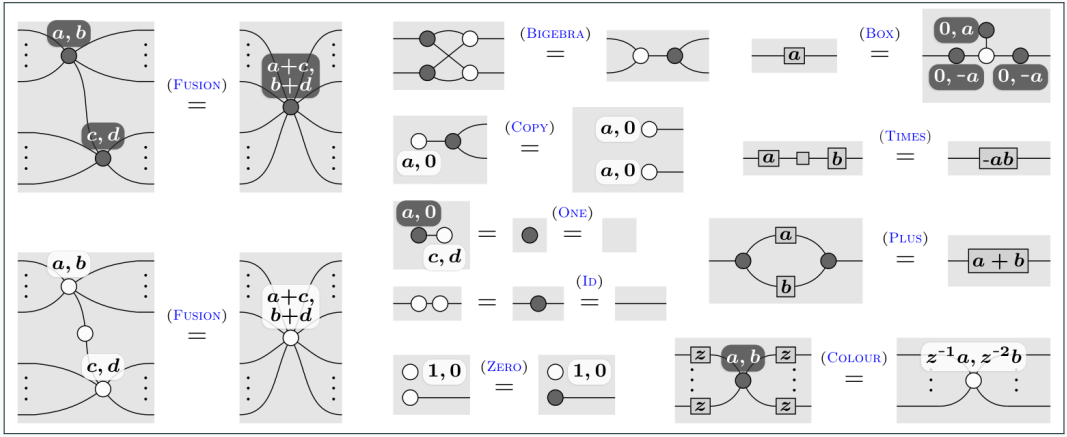


Boldizsár Poór et al. **"The Qupit Stabiliser ZX-travaganza: Simplified Axioms, Normal Forms and Graph-Theoretic Simplification".** In: *Electronic Proceedings in*

Stabilisers in the stabiliser ZX-calculus in prime dimensions



Graphical symplectic algebra



Theorem. $\mathsf{GSA}_{\mathbb{F}} \cong \mathsf{AffLagRel}_{\mathbb{F}}$

Robert I. Booth, Titouan Carette, and Cole Comfort. **Graphical Symplectic Algebra.** Jan. 30, 2024.

Continuous-variable quantum computing

Hilbert space of square-integrable functions:

$$\int_{x \in \mathbb{R}} |f(x)|^2 \leqslant +\infty.$$

$\mathrm{GSA}_{\mathbb{R}}$ has all the the features we want:

- it describes a stabiliser theory for functions on $\mathbb{R}$;
- it's compact closed
- it has a simple presentation in terms of generators and equations.

Thanks for listening...

But...

Problem:

$$\left[\!\left[\boxed{x, 0} \right]\!\right] \notin L^2(\mathbb{R})$$

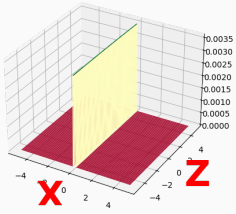
Physicists uses states like $\boxed{x, 0}$ and  all the time in arguments...

Question: Design a graphical language that both

- has the nice features of this CV "stabiliser" theory; and,
- captures a physically meaningful fragment of CV quantum mechanics.

Gaussian quantum mechanics in phase-space

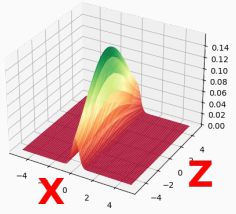
Dirac delta distribution



convolution by

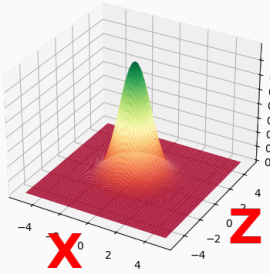
$$\mathcal{N}\left(\begin{bmatrix}\varepsilon & 0\\ 0 & \varepsilon^{-1}\end{bmatrix}, \begin{bmatrix}0\\ 0\end{bmatrix}\right)$$

Gaussian density function



The vacuum

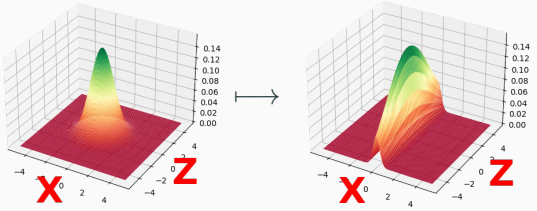
The **quantum vacuum state** is represented by the Gaussian distribution



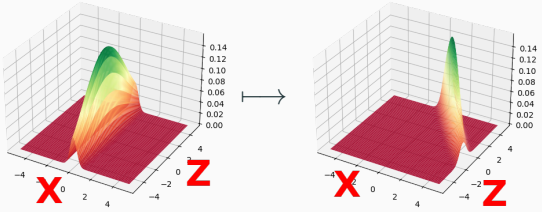
$$\rightsquigarrow \mathcal{N}\left(\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}, \begin{bmatrix}0\\ 0\end{bmatrix}\right)$$

Other Gaussians from the vacuum

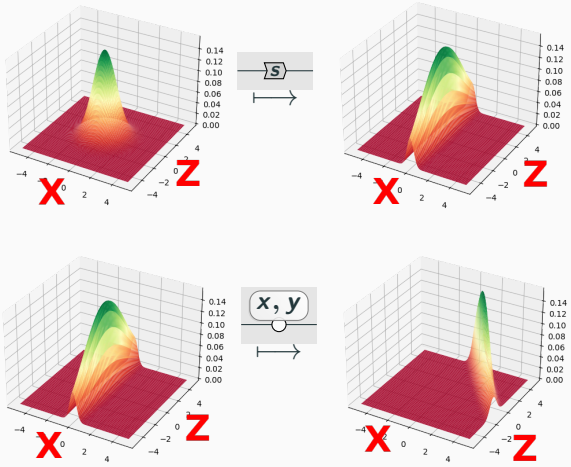
We can squeeze Gaussian distributions:



as well as displace and rotate them:



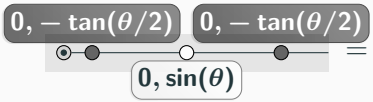
Other Gaussians from the vacuum



Characterising the vacuum

Theorem
The vacuum state can be freely added to $\mathsf{GSA}_{\mathbb{R}}$ *as a generator* $\circledcirc\!-$*, such that for all* $\vartheta \in [0, 2\pi)$ *and* $\theta \in (-\pi, \pi)$*:*

=



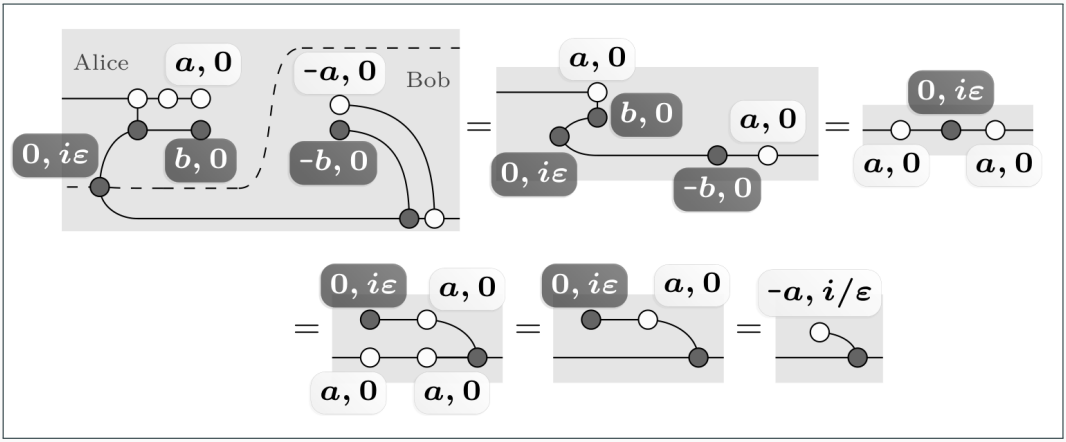
=

rotation of $\mathbb{R}^2$ *by* θ
decomposed into three shears

=

rotation of $\mathbb{R}^2$ *by* ϑ*, embedded along* $\mathsf{AffRel}_{\mathbb{R}} \hookrightarrow \mathsf{AffLagRel}_{\mathbb{R}}$

CV quantum teleportation



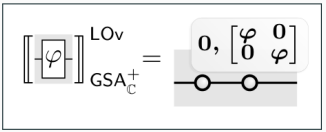
Conclusion

- first completeness result for an infinite-dimensional fragment of quantum theory;
- simple, relational semantics;

Teaser

There are many more things in the paper:

(a) Classical Gaussians



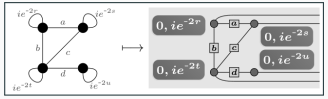
(b) LOv-calculus

$[\![\mathrm{QGauss}]\!] = \mathit{CPM}([\![\mathrm{GSA}_{\mathbb{R}}]\!])$

(c) Semantics

$\mathrm{QGauss} \to \mathrm{GSA}_{\mathbb{C}}$

(d) Complex embedding



(e) Graph representation

$\mathbf{0}, U + iV$

(f) Scalable notation